TOPICS COVERED IN THIS LECTURE

- **Numerical (FEM) techniques used in stability problems:**

  Use of FEM techniques to solve nonlinear problems in solid mechanics and to detect critical load and corresponding modes

- **Stability problems involving different scales:**

  a) Fiber-reinforced composites

  In microstructured solids, instability phenomena starting at the microscopic scale (local instability, typically a bifurcation that destroys symmetry) do show up at the macroscopic level (global instability, typically in the form of a localization of the deformation pattern). Of interest is when (i.e. at what loads) the local instability starts and if it leads to catastrophic failure

FEM TECHNIQUES FOR SOLVING STABILITY PROBLEMS

• Using the Finite Element Method (FEM), problem's principal solution at load λ is obtained using an incremental Newton-Raphson algorithm. Method, which starts from zero load – zero displacement, always converges for adequately small load step $\Delta\lambda$ and gives equilibrium solution to any accuracy required.

• At any given load λ, method automatically calculates corresponding stability operator, which is the tangent stiffness matrix K(λ).

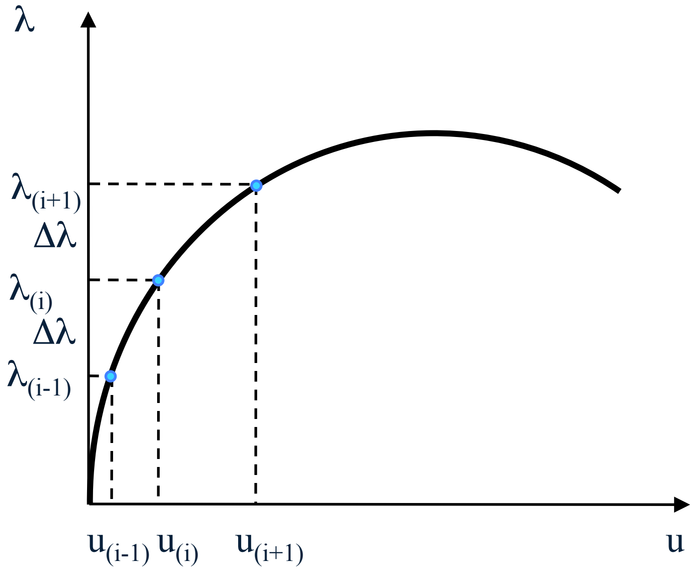
• Solution technique uses K = LDU (Cholesky) decomposition. Presence of critical points (i.e. limit load/bifurcation point) on the principal solution imply zeros in the diagonal matrix D (i.e. $(D_{ii})_{min}(\lambda_c) = 0$).

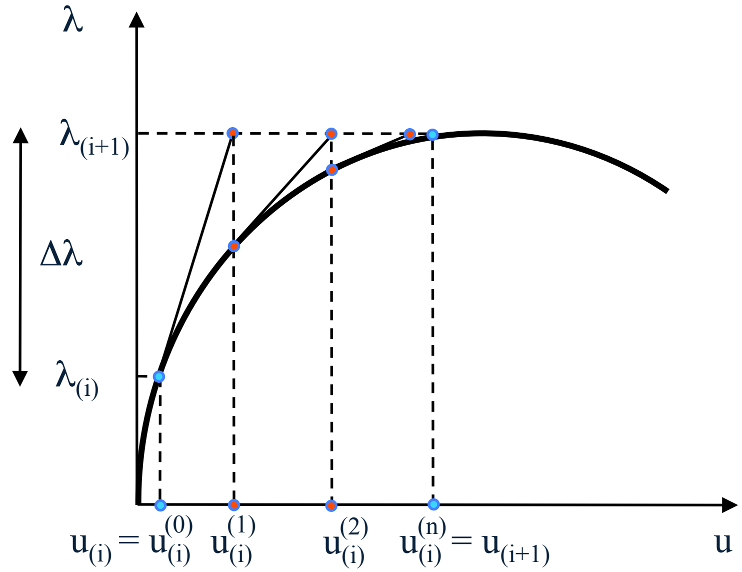
• Bisection method is used to accurately find the critical load.

• If the critical load is an m-tuple bifurcation point, the m lowest entries of D are zero at the critical load.

• If the critical load is a limit point, arc-length continuation methods are used to go past this point.

INCREMENTAL NEWTON-RAPHSON METHOD





Equilibrium solutions (principal/ bifurcated) of continuum problems amenable to finite d.o.f. case through discretization techniques (FEM)

Incremental method starting from zero load/displacement is needed to guarantee convergence of N-R (for small enough load steps)

N-R method finds equilibrium solutions and stability operator K.

K=LDU decomposition of stability operator – available as part of the solution procedure – in combination with bisection gives critical points

LSK asymptotics used to start bifurcated paths

FEM DISCRETIZATION OF NONLINEAR EUILIBRIUM

$\mathcal{E}(u, \lambda)$: continuum energy at displacement $u(\mathbf{x}) \in U$ and load $\lambda \geq 0$

$\mathcal{E}(\mathbf{u}, \lambda)$: discretized energy at displacement $\mathbf{u} \in \mathbb{R}^n$ and load $\lambda \geq 0$

$\mathbf{u} = \{u_i\}_{i=1}^{n}$: $u(\mathbf{x}) = \sum_{i=1}^{n} u_i \varphi_i(\mathbf{x}),\ u_i$: d.o.f, $\varphi_i(\mathbf{x}) \in U$: basis function

F.E.M. method : $\varphi_i(\mathbf{x})$ have compact support, by using element shape functions

ADVANTAGE : $\mathcal{E}_{,\mathbf{uu}}$ is banded matrix, i.e. populated about its diagonal

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$\mathcal{E}_{,\mathbf{u}}(\mathbf{u}, \lambda) = \mathbf{0}$: $\partial\mathcal{E}/\partial u_i = 0,\ i = 1 \ldots n$: equilibrium equations

Start at : $\lambda = 0, \quad \mathbf{u} = \mathbf{0}$

Continue by : Incremental Newton – Raphson Method

INCREMENTAL NEWTON-RAPHSON METHOD

$$\text{Newton} - \text{Raphson method}: \ \mathbf{0} = \mathcal{E}_{,\mathbf{u}}(\mathbf{u}+\Delta\mathbf{u},\lambda) \approx \mathcal{E}_{,\mathbf{u}}(\mathbf{u},\lambda) + \mathcal{E}_{,\mathbf{uu}}(\mathbf{u},\lambda)\Delta\mathbf{u} \ \Longrightarrow$$

$$\mathbf{u}_{(i)}^{(1)} - \mathbf{u}_{(i)}^{(0)} = -[\mathcal{E}_{,\mathbf{uu}}(\mathbf{u}_{(i)}^{(0)},\lambda_{(i)})]^{-1}\mathcal{E}_{,\mathbf{u}}(\mathbf{u}_{(i)}^{(0)},\lambda_{(i+1)}),$$

$$\text{start at } \mathbf{u}_{(i)}^{(0)} = \mathbf{u}_{(i)} \equiv \mathbf{u}(\lambda_{(i)}), \ \text{where } \mathbf{u}_{(i)}^{(j)}: \ \mathbf{u} \text{ at increment } (i) \text{ and iteration } (j)$$

$$\mathbf{u}_{(i)}^{(j+1)} - \mathbf{u}_{(i)}^{(j)} = -[\mathcal{E}_{,\mathbf{uu}}(\mathbf{u}_{(i)}^{(j)},\lambda_{(i+1)})]^{-1}\mathcal{E}_{,\mathbf{u}}(\mathbf{u}_{(i)}^{(j)},\lambda_{(i+1)}),$$

$$\text{end at } \mathbf{u}_{(i)}^{(j+1)} = \mathbf{u}_{(i+1)} \equiv \mathbf{u}(\lambda_{(i+1)}), \ \text{if error is small}: \ \|\mathcal{E}_{,\mathbf{u}}(\mathbf{u}_{(i)}^{(j)},\lambda_{(i+1)})\| < \varepsilon$$

STABILITY CHECK : positive definiteness of $\mathcal{E}_{,\mathbf{uu}} = \mathbf{LDU}$ (Choleski decomposition)

$\mathbf{L}$: lower triangular, $\mathbf{L}^T = \mathbf{U}$: upper triangular, $\mathbf{D}$: diagonal, matrices

$$\mathcal{E}_{,\mathbf{uu}}(\mathbf{u}(\lambda),\lambda) \text{ positive definite at load } \lambda \iff D_{ii}(\lambda) > 0, \ \forall\, 1 \le i \le n$$

GEOMETRIC STIFFNESS METHOD FOR CRITICAL LOAD

• Several FEM codes include a method termed "linear buckling" analysis or "geometric stiffness method" which is based upon the approximation of the stability operator, typically about the initial, stress-free configuration.

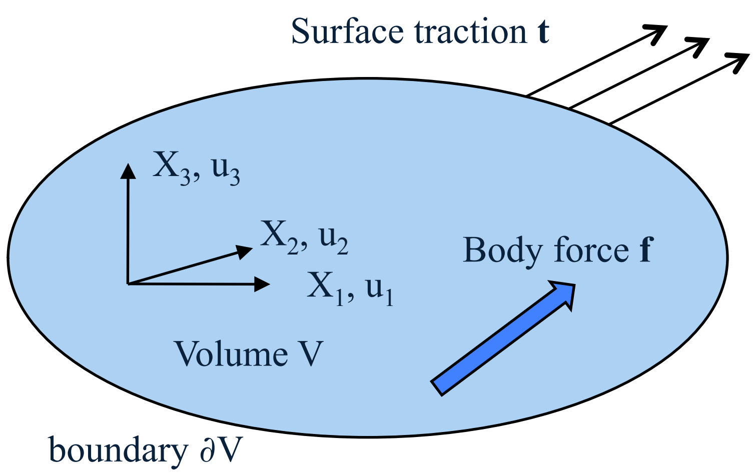
• The method works for linearly elastic structures with small strains and moderate rotations and seeks a multiplier of the stress state that will change the positive definiteness of the approximate stability operator.

• The stiffness matrix of the FEM discretized structure is approximated as: $\mathbf{K}_e + \lambda\mathbf{K}_g$, where K_e is the elastic stiffness matrix and K_g is the geometric stiffness matrix, both independent on the solid or structure's current geometry.

• The critical load λ_c is found from: $\mathrm{Det}\,[\mathbf{K}_e + \lambda_c\mathbf{K}_g] = 0$.

NOTE: Use caution when you apply this method, especially for solids/structures with material nonlinearities and large deformations.

GEOMETRIC STIFFNESS METHOD FOR CRITICAL LOAD



$$\mathcal{E}(u,\lambda) = \mathcal{E}_{int} + \mathcal{E}_{ext},$$

Displacement : $u = \mathbf{u}(\mathbf{X}) = (u_1(\mathbf{X}), u_2(\mathbf{X}), u_1(\mathbf{X})),$

At reference configuration point : $\mathbf{X} = (X_1, X_2, X_3)$

Elastic energy density/ ref. volume : $W(\mathbf{E}(\mathbf{X}), \mathbf{X})$

Lagrangian strain : $E_{ij} \equiv \frac{1}{2}(u_{i,j} + u_{j,i} + u_{k,i}u_{k,i})$

$$\mathcal{E}_{int} = \int_V W(\mathbf{E}(\mathbf{X}), \mathbf{X})dV, \quad \mathcal{E}_{ext} = -\left[\int_V f_i(\lambda)u_i dV + \int_{\partial V} t_i(\lambda)u_i dA\right]; \quad u_{i,j} \equiv \partial u_i/\partial X_j$$

$$W(\mathbf{0}, \mathbf{X}) = 0, \quad \left[\frac{\partial W}{\partial E_{ij}}\right]_{\mathbf{E}=\mathbf{0}} = 0, \quad \left[\frac{\partial^2 W}{\partial E_{ij}\partial E_{kl}}\right]_{\mathbf{E}=\mathbf{0}} = L^e_{ijkl} : \text{ elastic moduli}$$

GEOMETRIC STIFFNESS METHOD FOR CRITICAL LOAD

$$(\mathcal{E},^{0}_{uu} \Delta u)\delta u = \int_V \left\{ \left[\frac{\partial^2 W}{\partial E_{ij} \partial E_{kl}} \right]_{\overset{0}{u}} \Delta E_{ij} \delta E_{kl} + \left[\frac{\partial W}{\partial E_{kl}} \right]_{\overset{0}{u}} \Delta u_{k,i} \delta u_{k,j} \right\} dV$$

$$\Delta E_{ij} = \frac{1}{2}(\Delta u_{i,j} + \Delta u_{j,i} + \Delta u_{k,i} \overset{0}{u}_{k,j} + \overset{0}{u}_{k,i} \Delta u_{k,j})$$

$$\delta E_{ij} = \frac{1}{2}(\delta u_{i,j} + \delta u_{j,i} + \delta u_{k,i} \overset{0}{u}_{k,j} + \overset{0}{u}_{k,i} \delta u_{k,j})$$

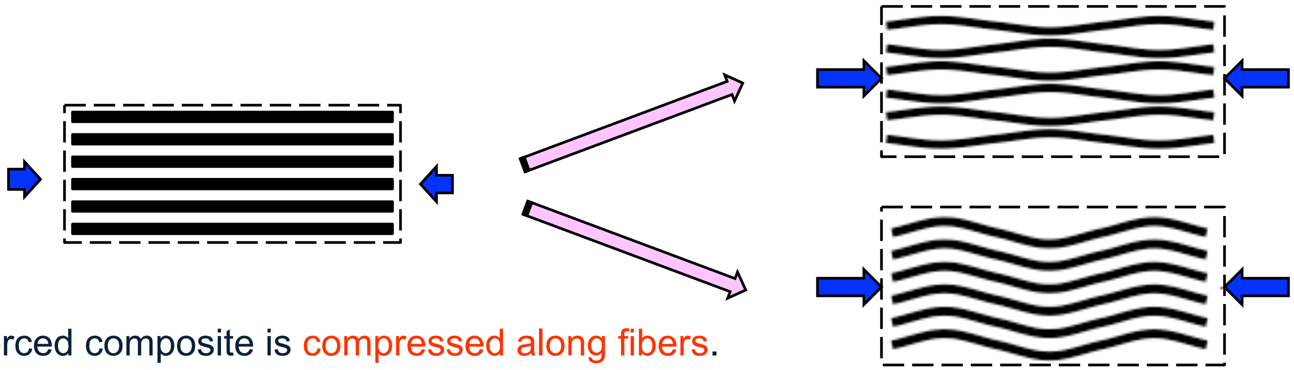
$$\text{Approximation :} \quad \left[\frac{\partial^2 W}{\partial E_{ij} \partial E_{kl}} \right]_{\overset{0}{u}} \approx L^e_{ijkl}, \quad \left[\frac{\partial W}{\partial E_{kl}} \right]_{\overset{0}{u}} \approx \lambda \overset{0}{\sigma}_{ij}, \quad \Delta E_{ij} \approx \frac{1}{2}(\Delta u_{i,j} + \Delta u_{j,i})$$

$$(\mathcal{E},^{0}_{uu} \Delta u)\delta u \approx \int_V \left[L^e_{ijkl} \Delta u_{i,j} \delta u_{k,l} + \lambda \overset{0}{\sigma}_{ij} \Delta u_{k,i} \delta u_{k,j} \right] dV \implies \text{F.E.M. discretization}$$

$$(\mathcal{E},^{0}_{uu} \Delta u)\delta u \approx \mathbf{\Delta u}^T \mathbf{K}(\lambda) \delta \mathbf{u} = \mathbf{\Delta u}^T [\mathbf{K}_e + \lambda \mathbf{K}_g] \delta \mathbf{u}$$

At $\lambda = 0$: $\mathbf{K}(0) = \mathbf{K}_e$ Pos. def.; Critical point is lowest λ_c : $\text{Det}[\mathbf{K}_e + \lambda_c \mathbf{K}_g] = 0$

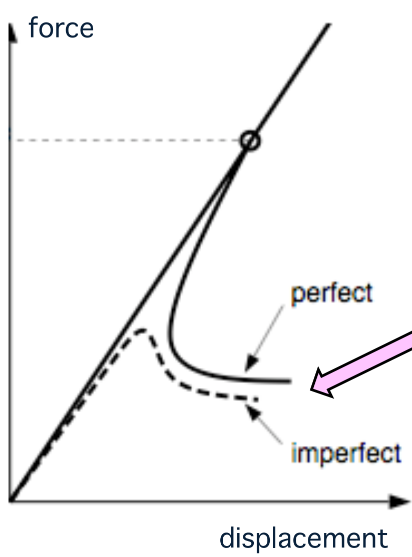
AXIAL COMPRESSION IN FIBER-REINFORCED COMPOSITES



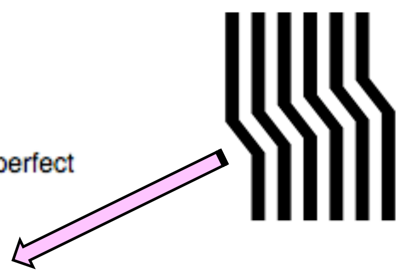
Local mode (short wavelength, dictated by microstructure)

Global mode (long wavelength, dictated by structure dimensions)

Fiber-reinforced composite is compressed along fibers.
When forces/displacements exceed a critical value:



Kink-band mode

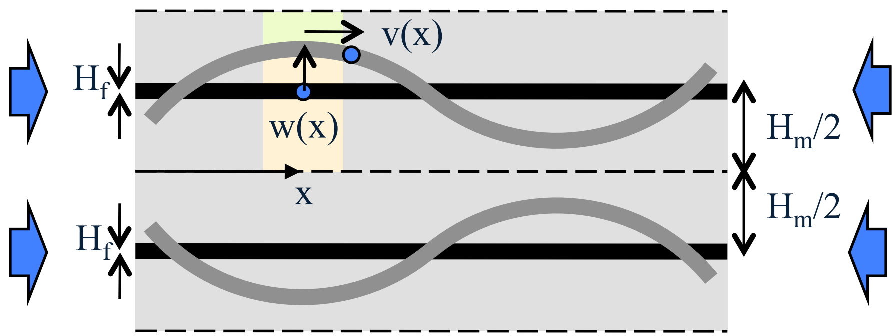


Graphite-epoxy

Balsa wood

Global mode is usually catastrophic, since it involves a post-bifurcated solution with a reduction of both force and displacement. Post-buckling deformed pattern involves kink-band modes.

AXIAL COMPRESSION - LOCAL FAILURE MECHANISM



For the case of thin, much stiffer than the matrix fibers, they buckle as beams on an elastic foundation. The foundation stiffness constant is $k = 2(E_m/0.5H_m)$ – since both the green and orange columns exert normal forces on the fiber due to the vertical displacement $w(x)$

$$\mathcal{E}_{int} = \int_x \frac{1}{2}[E_f H_f \epsilon^2 + E_f \frac{H_f^3}{12}\kappa^2 + 4\frac{E_m}{H_m} w^2]dx;\ \text{fiber (axial + bending) + matrix energy}$$

$$\text{fiber axial strain} : \epsilon = v,_x + \frac{1}{2}(w,_x)^2,\ \text{fiber bending strain} : \kappa = -w,_{xx};\ (\text{here } f,_x \equiv \frac{df}{dx})$$

$$\mathcal{E}_{ext} = \lambda E_f H_f[v(L) - v(0)] = \int_x [\lambda E_f H_f v,_x]\,dx, \quad \lambda = -\overset{0}{v},_x :\ \text{imposed axial strain}$$

$$\mathcal{E}(u,\lambda) = \mathcal{E}_{int} + \mathcal{E}_{ext} = \int_x \{\frac{1}{2}[E_f H_f \epsilon^2 + E_f \frac{H_f^3}{12}\kappa^2 + 4\frac{E_m}{H_m} w^2] + \lambda E_f H_f v,_x\}dx;\ u \equiv (v, w)$$

AXIAL COMPRESSION - LOCAL FAILURE MECHANISM

$$\mathcal{E},_u = \int_x [E_f H_f \epsilon \delta\epsilon + E_f \frac{H_f^3}{12} \kappa \delta\kappa + 4\frac{E_m}{H_m} w \delta w + \lambda E_f H_f \delta v,_x] dx = 0,$$

where : $\delta\epsilon = \delta v,_x + w,_x \delta w,_x \,, \quad \delta\kappa = -\delta w,_{xx} \,,$

Principal solution (straight configuration) : $\epsilon_0(x) = \overset{0}{v},_x(x) = -\lambda, \quad \overset{0}{w}(x) = 0.$

$$\mathcal{E},^c_{uu} = \int_x [E_f H_f \overset{1}{v},_x \delta v,_x + E_f H_f \epsilon_0 \overset{1}{w},_x \delta w,_x + E_f \frac{H_f^3}{12} \overset{1}{w},_{xx} \delta w,_{xx} + 4\frac{E_m}{H_m} \overset{1}{w} \delta w] dx = 0,$$

term δv : $(E_f H_f \overset{1}{v},_x),_x = 0 \quad \Longrightarrow \quad \overset{1}{v},_x(x) = 0,$

term δw : $E_f H_f \lambda_c \overset{1}{w},_{xx} + E_f \frac{H_f^3}{12} \overset{1}{w},_{xxxx} + 4\frac{E_m}{H_m} \overset{1}{w} = 0 \quad \Longrightarrow \quad \overset{1}{w}(x) = \alpha \sin(\omega x).$

NOTE : bifurcation at λ_c since $\mathcal{E},_{u\lambda} \overset{1}{u} = \int_x [E_f H_f \overset{1}{v},_x] dx = 0.$

AXIAL COMPRESSION - LOCAL FAILURE MECHANISM

$$\text{critical load}: \lambda_c = \min_{\omega H_f \in \mathbb{R}} \left[\frac{(\omega H_f)^4/12 + 4(E_m H_f / E_f H_m)}{(\omega H_f)^2} \right], \text{ at}: (\omega_c H_f)^4 = 48 \frac{E_m H_f}{E_f H_m}.$$

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volume fract. fiber : $V_f = H_f/H$, volume fract. matrix : $V_m = H_m/H$, $H \equiv H_f + H_m$.

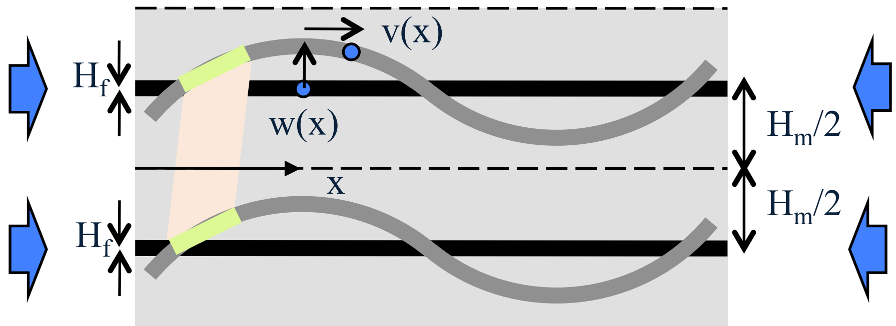
$$\boxed{\lambda_c = 2 \left[\frac{E_m V_f}{3E_f(1 - V_f)} \right]^{1/2}};$$ critical strain for local buckling mode.

$$\boxed{\frac{L_c}{H} = \pi V_f \left[\frac{E_f(1 - V_f)}{3E_m V_f} \right]^{1/4}};$$ critical dimensionless wavelength for local buckling mode.

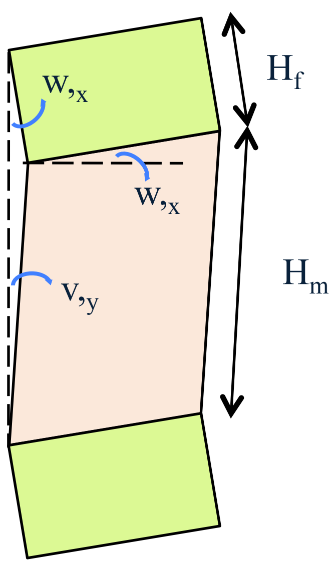
EXAMPLE (typical of graphite fiber, epoxy matrix) : $E_f/E_m = 100$.

$\lambda_c \approx 3.8\%$, $L_c \approx 1.31H = 13.1H_f$, for : $V_f = 10\%$.

AXIAL COMPRESSION – GLOBAL FAILURE MECHANISM



For the case of thick, much stiffer than the matrix fibers, they rotate as to remain parallel to each other. The matrix shear strain is $\gamma = (v_{,y} + w_{,x})$ – and energy is stored in axial deformation of fiber (no bending) and in shear deformation of matrix.



$$\mathcal{E}_{int} = \int_x \frac{1}{2}[E_f H_f \epsilon^2 + G_m H_m \gamma^2]dx; \text{ (fiber axial + matrix shear)}$$

$$\text{fiber axial} : \epsilon = v,_x + \frac{1}{2}(w,_x)^2, \text{ matrix shear} : \gamma = v,_y + w,_x ,$$

$$\text{Notice from kinematics (small angles)} : \; H_m v,_y = H_f w,_x$$

$$\mathcal{E}_{ext} = \lambda E_f H_f [v(L) - v(0)] = \int_x [\lambda E_f H_f v,_x]\, dx,$$

$$\mathcal{E}(u,\lambda) = \mathcal{E}_{int} + \mathcal{E}_{ext} = \int_x \{\frac{1}{2}[E_f H_f \epsilon^2 + G_m H_m \gamma^2] + \lambda E_f H_f v,_x\}dx$$

AXIAL COMPRESSION – GLOBAL FAILURE MECHANISM

$$\mathcal{E}_{,u} = \int_x [E_f H_f \epsilon \delta\epsilon + G_m H_m \gamma \delta\gamma + \lambda E_f H_f \delta v_{,x}] dx = 0,$$

where : $\delta\epsilon = \delta v_{,x} + w_{,x} \delta w_{,x}$, $\delta\gamma = \delta w_{,x} (H_f + H_m)/H_m = \delta w_{,x} / V_m$

Principal solution (straight configuration) : $\boxed{\epsilon_0(x) = \overset{0}{v}_{,x}(x) = -\lambda, \quad \overset{0}{w}(x) = 0.}$

$$\mathcal{E}^{c}_{,uu} = \int_x [E_f H_f \overset{1}{v}_{,x} \delta v_{,x} + E_f H_f \epsilon_0 \overset{1}{w}_{,x} \delta w_{,x} + G_m \frac{H_m}{V_m^2} \overset{1}{w}_{,x} \delta w] dx = 0,$$

term δv : $(E_f H_f \overset{1}{v}_{,x})_{,x} = 0 \quad \Longrightarrow \quad \boxed{\overset{1}{v}_{,x}(x) = 0,}$

term δw : $E_f H_f \lambda_c \overset{1}{w}_{,xx} - G_m \frac{H_m}{V_m^2} \overset{1}{w}_{,xx} = 0 \quad \Longrightarrow \quad \boxed{\overset{1}{w}_{,xx}(x) \neq 0.}$

NOTE : bifurcation point at λ_c since $\mathcal{E}_{,u\lambda} \overset{1}{u} = \int_x [E_f H_f \overset{1}{v}_{,x}] dx = 0.$

AXIAL COMPRESSION – GLOBAL FAILURE MECHANISM

critical load : $\lambda_c = \dfrac{G_m}{E_f V_f V_m} = \dfrac{E_m}{2E_f(1+\nu_m)V_f(1-V_f)}$

NOTE : $G_m = E_m/[2(1+\nu_m)]$, ν_m : Poisson ratio of matrix.

EXAMPLE (typical of graphite fiber, epoxy matrix) : $E_f/E_m = 100$, $\nu_m = 1/4$.

$\lambda_c \approx 4.4\%$, for : $V_f = 10\%$ Thin fibers, global mode found AFTER local.

For thick fibers : $V_f = 33\%$, global mode : $\lambda_c = 1.8\%$, local mode : $\lambda_c = 8.2\%$

NOTE : for layered composites in plane strain replace E_f by $E_f/(1-\nu_f)^2$

AXIAL COMPRESSION – NUMERICAL (FEM) CALCULATIONS

- Calculations for axially compressed, fiber-reinforced composites done using a hyperelastic, layered composite (Mooney-Rivlin) in plane strain (finite strain formulation is used to solve this FEM problem).

- Finite strain formulation needed because it takes consistently into account kinematic as well as constitutive nonlinearities. Hyperelastic material is the simplest model with these properties.

$$\boxed{W(\mathbf{E}) = A(I_1 - 3) + B(I_2 - 3),\ I_3 = 1,}\ A, B > 0;\ \text{incompressible Mooney} - \text{Rivlin}$$

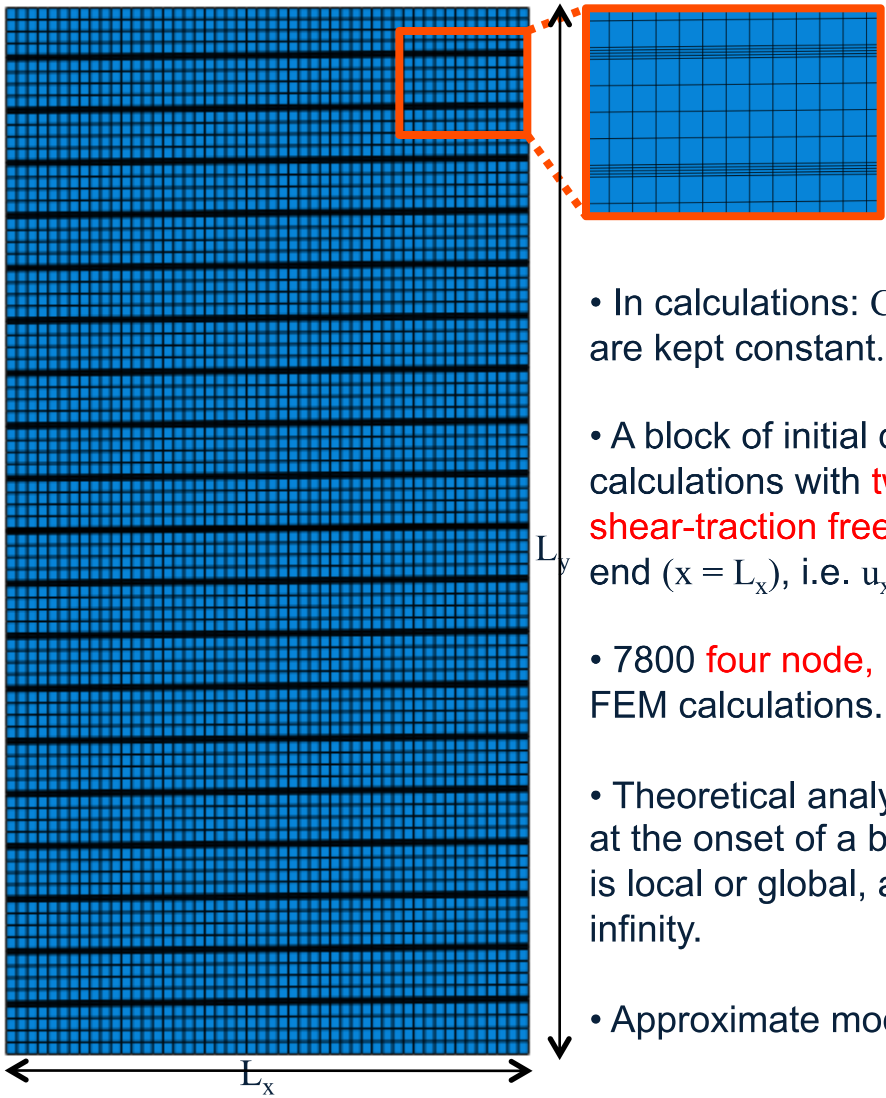
$$I_1 = \mathrm{tr}(\mathbf{C}),\ I_2 = \frac{1}{2}[(\mathrm{tr}(\mathbf{C})^2 - \mathrm{tr}(\mathbf{C}^2)],\ I_3 = \mathrm{Det}(\mathbf{C});\ \text{invariants of Cauchy} - \text{Green } \mathbf{C}$$

$$\mathbf{E} = \frac{1}{2}(\mathbf{C} - \mathbf{I})\ \text{Green} - \text{Lagrange strain},\ \mathbf{C} = \mathbf{F}^T \bullet \mathbf{F},\ \text{deformation gradient } \mathbf{F} = \mathbf{I} + \mathbf{u}\nabla$$

$$\boxed{W(\mathbf{E}) = A(\frac{I_1}{I_3^{1/3}} - 3) + B(\frac{I_2}{I_3^{2/3}} - 3) + C(I_3^{1/2} - 1)^2,}\ C >> A, B > 0;\ \text{compressible case}$$

compressible Mooney – Rivlin used in FEM (plus underintegration or special elements)

AXIAL COMPRESSION – NUMERICAL (FEM) CALCULATIONS



- Calculations for axially compressed, fiber-reinforced composites done using a hyperelastic, layered composite (Mooney-Rivlin) in plane strain (finite strain formulation is used to solve this FEM problem).

- In calculations: $C \gg A = 4B$ and the ratios $A_f/A_m = B_f/B_m = C_f/C_m = E_f/E_m$ are kept constant.

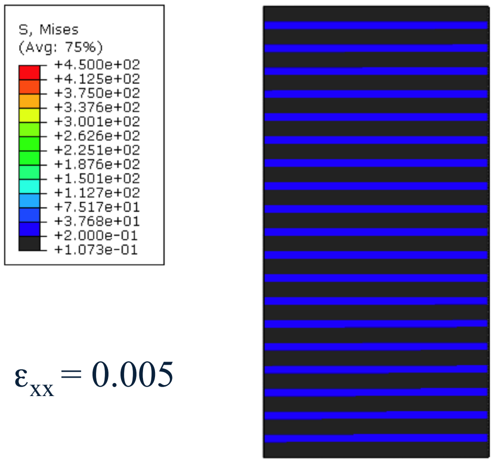
- A block of initial dimensions $L_x \times L_y$ (where $L_y = 2L_x$) is used for the calculations with two ends free ($y = 0$, $y = L_y$) and two ends straight and shear-traction free ($x = 0$, $x = L_x$) with a displacement imposed on the right end ($x = L_x$), i.e. $u_x(L_x) = -\Delta$). The thickness of each unit cell is H.

- 7800 four node, bilinear quadrilateral elements are used in the reported FEM calculations. Model has 20 layers of matrix and 19 fibers.

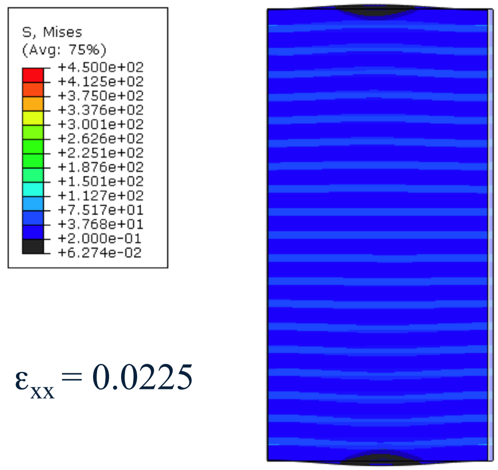
- Theoretical analysis (exact analytical model) can predict critical strain λ_c at the onset of a bifurcation instability, and also determine if the instability is local or global, according to if the wavelength $(L/H)_c$ is finite or goes to infinity.

- Approximate model is reasonable for large values of E_f/E_m.

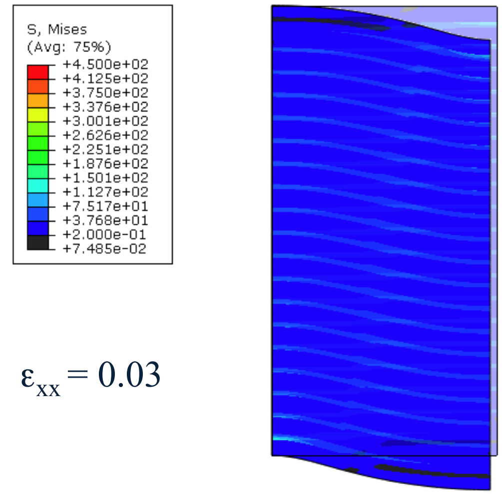
AXIAL COMPRESSION – GLOBAL FAILURE MECHANISM



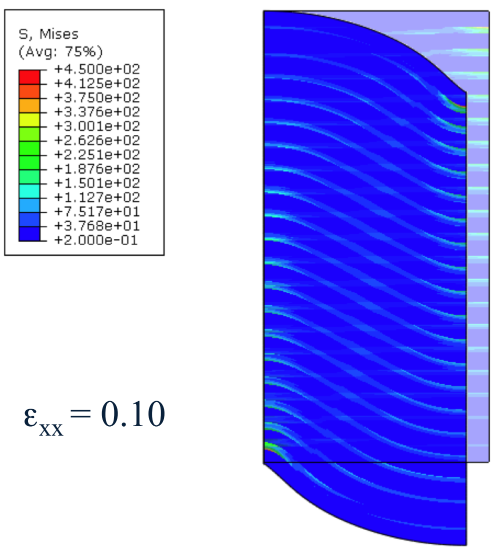
$\varepsilon_{xx} = 0.005$



$\varepsilon_{xx} = 0.0225$



$\varepsilon_{xx} = 0.03$

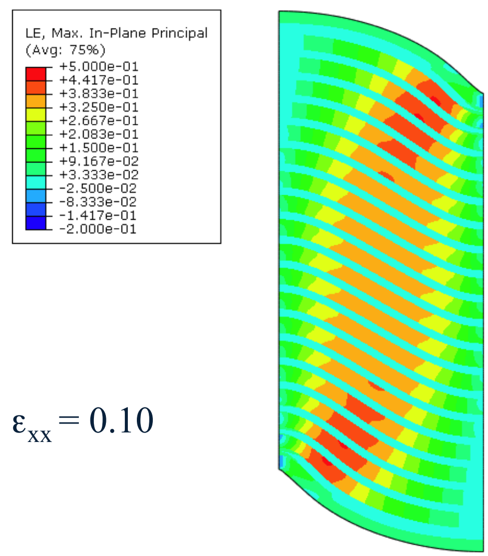


$\varepsilon_{xx} = 0.10$

- Calculations for thick fiber composite:
- $E_f/E_m = 100$, $V_f = 0.33$.

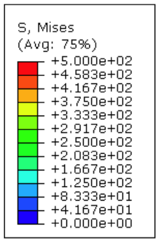
- Imperfection used: all fibers were tilted with respect to their perfect position by an angle $\phi = 0.001$ (same for all fibers)

- Notice symmetric (bareling) mode at $\varepsilon_{xx} = 0.0225$ before going to antisymmetric (S-shape) mode for higher strains.

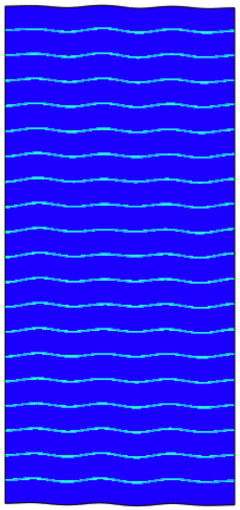


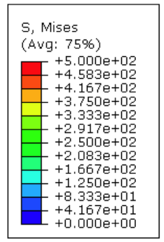
$\varepsilon_{xx} = 0.10$

AXIAL COMPRESSION – LOCAL FAILURE MECHANISM

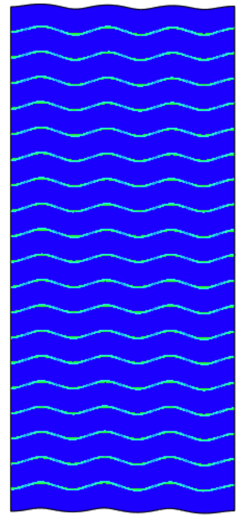


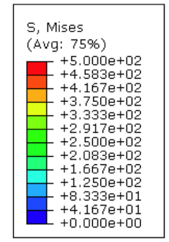
$\varepsilon_{xx} = 0.05$



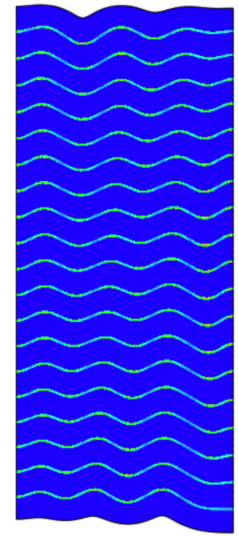


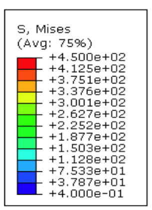
$\varepsilon_{xx} = 0.07$



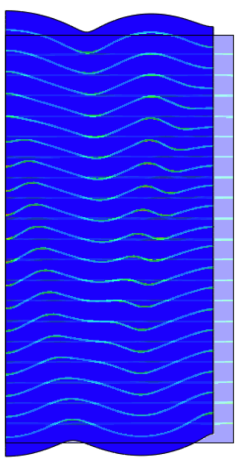


$\varepsilon_{xx} = 0.10$





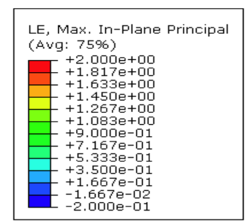
$\varepsilon_{xx} = 0.09$



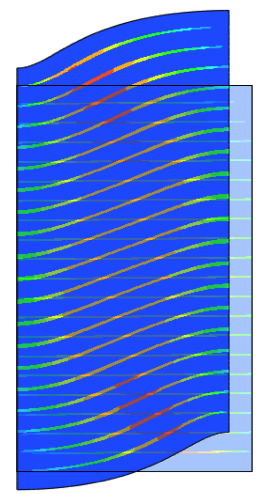
- Calculations for thin fiber composite: $E_f/E_m = 100$, $V_f = 0.10$.

- Imperfection in top 3 figs: fibers were tilted with respect to their perfect position by a random angle $0.001 < \phi < 0.003$.

- Imperfection in left fig: fibers were tilted by a random angle $-0.001 < \phi < 0.003$, while at right fig. all fibers have same tilt $\phi = 0.002$.



$\varepsilon_{xx} = 0.10$



AXIAL COMPRESSION IN FIBER-REINFORCED COMPOSITES

• Fiber reinforcing of solids is good for tension, but bad for compression due to instability.

• Instability of fiber-reinforced composites under compression due primarily to nonlinear kinematics.

• Approximate models presented find critical load and mode for high fiber/matrix stiffness ratios.

• Thin, stiff fibers correspond to a local critical mode, thick fibers to a global critical mode.

• Exact, analytical solution exists for nonlinear layered solids (finite strain, nonlinear constitutive response) to determine critical load and wavelength of corresponding mode (local or global).

• Post-buckling behavior requires numerical solution.

• Careful in seeking numerical solutions because (depending on the imperfection you put) you might miss modes.

• NOTE: Having analytical solutions/asymptotic results available is always a good idea when doing numerical solutions of nonlinear problems to avoid mistakes…