Example file for the `MultiLayerIndentation` Package

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Source for the `MultiLayerIndentation` Package

A. Constantinescu and A.M. Korsunsky - Elasticity with Mathematica (r)
   Cambridge University Press, 2007

A.M. Korsunsky, A. Constantinescu - The influence of indenter bluntness on the apparent contact stiffness of thin coatings,
   Thin Solid Films 517 (2009) 4835 4844

A. Constantinescu, A.M. Korsunsky, O. Pison, A. Oueslati - Symbolic and numerical solution of the axisymmetric indentation problem of a multilayered elastic solid
   submitted to Int.J. Solids and structures, 2013

H.Y. Yu, S.C. Sanday, B.B. Rath, The effect of substrate on the elastic properties of films determined by the indentation test — axisymmetric Boussinesq problem,
   J. Mech. Phys. Solids 38 (6) (1990) 745.

N.N. Lebedev, I.S. Uflyand, Prikladnaya Matematica Mehanica 22 (1958) 320

- **Load the Package**

```
Directory[]
```

```
/mydata/LMS/Indentation/Indentation_Olivier/IndentationPackage
```

```
Clear["Global`*"]
<< MultiLayerIndentation.m
```

The functions of the package : MultiLayerIndentation`

Click on the buttons below to get the usage of the indentation functions

```
? *Indentation
```

▼ MultiLayerIndentation`

MultiLayerIndentation	OneLayerIndentation

Click on the buttons below to get the usage of the additional functions of the package used for numerical integration using Gauss - points and weights

```
? MultiLayerIndentation`Gauss*
```

▼ MultiLayerIndentation`

GaussIntegrate	GaussInterval	GaussStuff	GaussStuffInterval

Indentation Examples

- **Indentation of a one layer elastic coating**

Definition of material data :

```
hsslide = {480, 0.2, 84, 0.4, "Slide"}
hsbound = {480, 0.2, 84, 0.4, "Bound"}
depths = N[Exp[Union[Range[-6, 0, 0.5], Range[0, 3.5, 0.25]]]];

punch = {"Cone", Tan[(90 - 70.3) Pi / 180]}
```

```
{480, 0.2, 84, 0.4, Slide}
```

```
{480, 0.2, 84, 0.4, Bound}
```

```
{Cone, 0.358052}
```

```
depths = N[Exp[Range[-6, 2, 0.05]]];
```
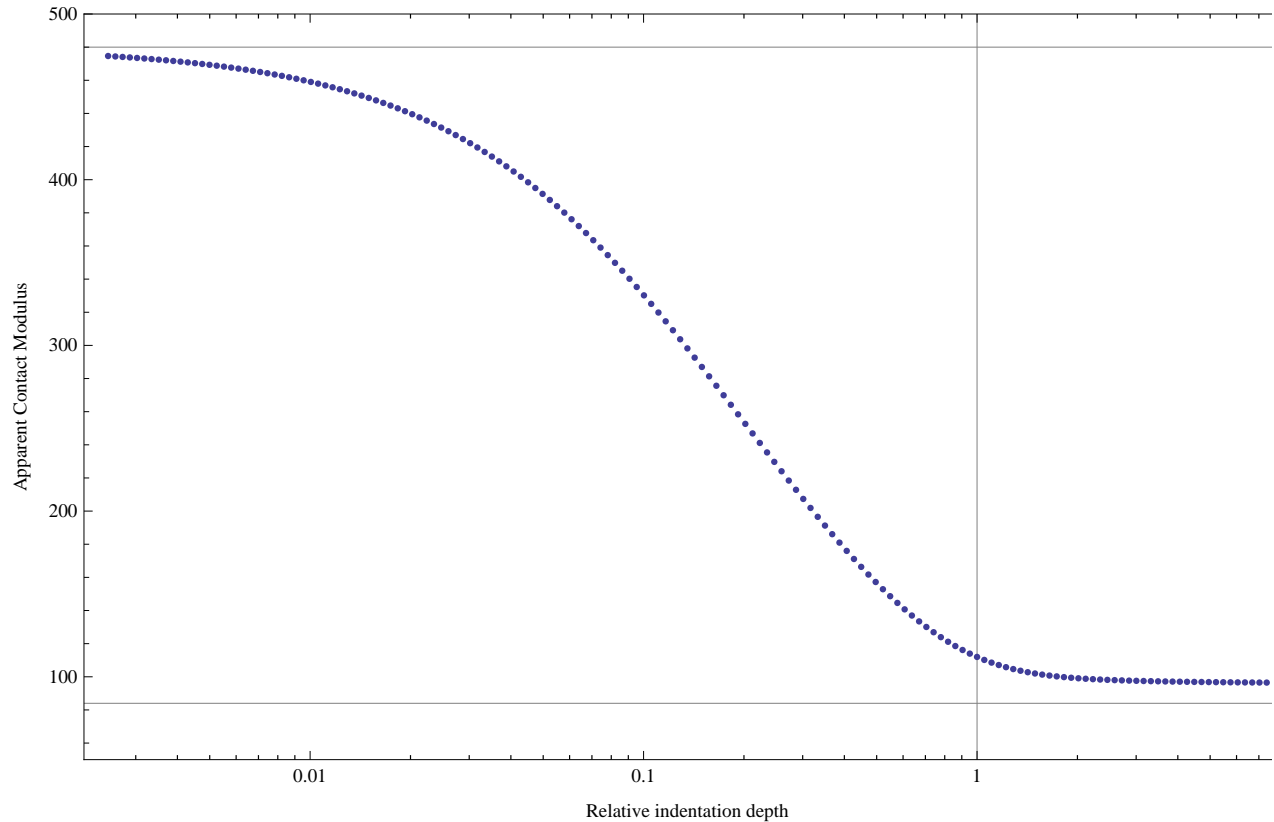
```
ndiscret = {25, 25};
```

```
Timing[l1 = OneLayerIndentation[ depths, punch, hsslide, ndiscret] ;]
```

```
{38.6091, Null}
```

```
l1dPStar = Map[ {#[[1]], #[[5]]} &, l1];

ListLogLinearPlot[l1dPStar, Frame -> True,
 GridLines -> {{1}, {84., 480}}, PlotRange → {All, {50, 500}},
 FrameLabel -> {"Relative indentation depth", "Apparent Contact Modulus"}]
```


■ Indentation of multilayer layer elastic coating

```
se = 187. × 10 ^ 9 ; sn = 0.28;
ee = {65. × 10 ^ 9, 277. × 10 ^ 9}; nn = {0.33, 0.17};
hh = {50. × 10 ^ -6, 30. × 10 ^ -6 };
lesE = Append[ Drop[ Flatten[ Array[ ee & , 3] ], -2], se];
lesNu = Append[ Drop[ Flatten[ Array[ nn & , 3] ], -2], sn];
lesmu = lesE / ( 1 + lesNu) / 2 ;
lesz = {1., 1.6, 2.6, 3.2 };
mat = {lesNu, lesmu, lesz}
```

$\{\{0.33, 0.17, 0.33, 0.17, 0.28\},$
$\ \{2.44361 \times 10^{10}, 1.18376 \times 10^{11}, 2.44361 \times 10^{10}, 1.18376 \times 10^{11}, 7.30469 \times 10^{10}\}, \{1., 1.6, 2.6, 3.2\}\}$

```
punch = {"Cone", Cot[(70.3) * Pi / 180]};
punchflat = {"Flat", 1.2*^-6};
punchsphere = {"Hertz", 2.*^5};
punchrock = {"Rock", Cot[(70.3) * Pi / 180], 0.01};


dmax = 14.; num1 = 100; incr = dmax / num1;
depl = Table[i, {i, incr, dmax, incr}];
ndiscret = {40, 40};
```

```
Timing[l4 = MultiLayerIndentation[depl , punch, mat, ndiscret] ;]
```

{446.268, Null}

```
l4dPStar = Map[ {#[[1]], 10^-9 #[[5]]} &, l4];
```

```
ListLogLinearPlot[l4dPStar, Frame -> True, PlotRange -> {40, 220},
 GridLines -> {{1}, {71., 187}},
 FrameLabel -> {"Relative indentation depth", "Apparent Contact Modulus"},
 ImageSize -> 600]
```