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Dissipation and fatigue damage

A unified method to treat fatigue damage

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The discussion of fatigue damage is generally separated in two domains: Low Cycle Fatigue (LCF) and High Cycle Fatigue (HCF). The transition zone concerns the case of limited endurance, whether the unlimited endurance is generally assimilated with HCF. Fatigue failure is the result of complex microscopic phenomena which occur under cyclic loading, however the principal mechanism responsible of the crack initiation, common to all domains, is the spatial extension of inelastic strains (plastic or viscous) in the grains due to motion of dislocations. The major difference between HCF and LCF regimes is that inelastic strains develop at the mesoscopic and LCF regimes is that inelastic strains develop at the mesoscopic respectively and macroscopic scale of the material. As the underlying mechanics is the same, there should be no reason to have distinct criteria in HCF and LCF. The objective of this paper is to give some further considerations based on dissipation towards a unified method to treat fatigue damage.

The discussion of fatigue damage is generally separated in two domains: Low Cycle Fatigue (LCF) and High Cycle Fatigue (HCF).The transition zone concerns the case of limited endurance, whether the unlimited endurance is generally assimilated with HCF.

Fatigue failure is the result of complex microscopic phenomena which occur under cyclic loading, however the common principal mechanism responsible for the crack initiation is the spatial extension of inelastic strains (plastic or viscous) produces by the movement of dislocations. The main difference between the LCF and HCF regimes is the scale where the development of inelastic strains can be observed. In HCF they are only present at the mesoscopic scale of the grains, whether in LCF they can be observed at both the macroscopic and the mesocsopic scale of the material.

If both damage mechanism in HCF and LCF are based on the same physical phenomena, philosophically, there should be no reason to have distinct criteria in these domains.

The objective of this paper is to indicate an alternative way with respect to classical fatigue methods based on dissipation which might permit to obtain a unified approach in fatigue in the future.

A first step towards this unifying objective consists in a re-interpretation in terms of mesoscopic dissipation of the critical plane approach used in HCF as initially proposed by Dang Van [1] and extended by Papadopoulos [2]. This criterion determines the domain of unlimited endurance and is based on the following hypothesis:

- the cyclic behaviour of the crystal follows a two stages evolution of the yield limit (Figure 1),
- the plastic criterion of the crystal follows a Schmid law and a meso-macro passage is linking the mechanical fields at both scales.

This is generally presented in terms of stress tensors. The unlimited lifetime is assured by reaching an elastic shakedown state at the mesoscopic level. Therefore the technique can easily be translated in terms of plastic work [3]. The first part of this paper will detail this passage and discuss and illustrate it with experimental results from the literature [4, 5].

The second step towards the unifying objective is focused in the LCF regime, where plastic work is an important damage parameter and can be directly related to dissipation. One can note that the rate of plastic work is defined in equation 1

$$d_p = \underline{\underline{\sigma}} : \underline{\underline{\varepsilon}}_p \qquad (1)$$

in an elastoplastic framework, in small deformations context. The direct relation between damage in this second regime was previously proven by the present authors in the case of industrial structures submitted to thermomechanical low cycle fatigue [6]. The second part of this paper will refine a theoretical framework permitting the link between dissipation and fatigue damage in LCF and the theoretical concepts will be compared to experimental results obtained from the literature [7-9].

Fatigue damage: a mechanical approach

Fatigue damage and mechanical behaviour. In order to determine the possible links between thermomechanical dissipation and fatigue damage we shall need a simple framework for the description of the principal mechanisms of metals fatigue.

The evolution of metallic grains under complex loadings towards damage initiation gives rise to a series of mechanism starting with: localised slip bands, local cyclic plasticity in Persistent Slip Bands (PSB), microcracking along PSB, coalescence of microcracks, etc., which are well known and already described in the literature. Based on these observations and on the Orowan's theory of metals fatigue [8], Dang Van [1] proposed a mechanical framework for determining HCF. His method has later been enriched by the work of Papadopoulos [2]. We shall explain next the main hypothesis of this framework.

The first period of the lifetime of a structure submitted to cyclic loading is characterized by the formation of localised slip bands in the grains oriented unfavourably with respect to loading directions. These slip bands, initially often supercificial, transform with the evolving cycles into PSB. With cyclic loading, this phenomenon becomes irreversible and an increasing number of the PSB can be observed. Then schematically, when the loading value is close to the fatigue limit, microcracks can appear along the PSB. Let us denote by N_1 the value of cycles up to this point. It is important to remark that N1 is small compared to the complete lifetime of the structure.

The second period, from N_1 up to N_2 cycles, will cover the growth and the beginning of the coalescence of those micro-cracks in the PSB. The coalescence of the grain defects into the initiation of a macrocrack covers the third period from N_2 up to N_3 cycles. The last period, covers the initiation of a macrocrack at N_3 cycles and the lasts up to the final failure after N_f cycles.

As a consequence we remark that these grain mechanism control the damaging mechanism and their mechanical description should be part of a complete fatigue analysis. We shall retain that the mechanical description of the damaging mechanisms has to be realised at the mesoscopic scale, the scale of the grains.

Most metallic materials such as steels, aluminium alloys or copper consist in aggregates of cubic crystals with randomly distributed orientations. Therefore at the macroscopic scale, the material can still be consider homogeneous and isotropic even if, locally, at the mesoscopic scale, this is not true. The local anisotropy and heterogeneity at the mesoscopic level conducts to the appearance of "misoriented grains" with respect to the loading which can plastify and later conduct to the crack initiation.

When the imposed loading gives rise to a stress range close to the fatigue limit, the imposed macroscopic stresses are weak when compared to the yield limit. The macroscopic plastic strains are then negligible. However, due to mesoscopic heterogeneity, misoriented grains are subject to various stress concentrations. They undergo plastic strain and as a consequence strain hardening due to the rise of the number of dislocations in the slip bands. This means that local stresses in misoriented grains exceed the yield limit during at least a part of each cycle.

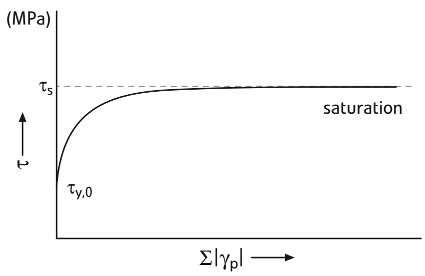

Figure 1. Schematic hardening curve of a mono crystal under cyclic loading. τ is the resolved shear stress acting on the gliding plane and $\tau_{y,0}$, the initial yield limit value. τ_s is the saturation stress and $\Sigma|\gamma_p|$ the cumulated plastic shear

One of the important hypothesis of the Dang Van approach in HCF, concerns the behaviour of weakest grains subjected to cyclic plasticity due to fatigue damage. Experiments conducted on monocrystals are interesting illustrations of the local mechanical behaviour of misoriented grains: the local stress amplitude is increasing in the early part of a fatigue test with constant strain amplitude. Then when a saturation stress (s is reached, this value will be maintained until a crack occurs. The grain will be completely cracked just some more cycles after this crack initiation (Figure 1).

Winter [9] showed on copper specimens not only that the PSBs start to form as the stress amplitude reaches its saturation value but also that this saturation stress is independent of the imposed plastic strain range. Moreover, observations of asymmetric hysteresis loops allow to conclude about a kinematic strain hardening process. The cyclic behaviour of misoriented grains has therefore to be represented by both isotropic and kinematic hardening laws.

Papadopoulos approach in HCF. At Dang Van's instigation, Papadopoulos [2] proposed a complete mechanical approach in HCF requiring a link between macro- and mesoscopic fields based on the previous classical observations of the fatigue damage process. Let us denote as $\underline{\underline{\Sigma}}$ and $\underline{\underline{\sigma}}$ the macroscopic and mesoscopic stress tensors and $\underline{\underline{E}}$ and $\underline{\underline{\varepsilon}}$ the macroscopic and mesoscopic strain tensors.

The link between the macro- and the mesoscopic scale can be obtained under different hypothesis. The Sachs model supposes the equality of stress tensors $\underline{\underline{\Sigma}}$ and $\underline{\underline{\sigma}}$ is especially suited for thin structures. For polycristalline materials it does not take correctly into account the misfit between the different grains. The Lin-Taylor model [10,11] supposes the equality of strain tensors $\underline{\underline{E}}$ and $\underline{\underline{\varepsilon}}$. The hypothesis is well suited for polycristalline materials, if the plastic strain is not spatially extended at the mesoscopic level. In this case we can model the reality considering a plastic inclusion in an infinite elastic matrix and the matrix imposes its deformation to the inclusion.

If the inclusion and the matrix are isotropic and have the same elastic properties, we obtain the relation in equation 2 between the macroscopic end mesoscopic stress tensors:

$$\underline{\underline{\sigma}} = \underline{\underline{\Sigma}} - 2\,\mu\underline{\underline{\varepsilon}}_p = \underline{\underline{\Sigma}} + \underline{\underline{p}} \qquad (2)$$

By $\underline{\underline{\varepsilon}}_p$ we have denoted the mesoscopic plastic strain tensor. μ represents the Lamé moduli and $\underline{\underline{\rho}}$ the mesoscopic residual stress tensor.

Taking into account experimental results, such as the ones presented by Winter [11], we can consider that a Representative Elementary Volume (REV) is composed by mean crystals governed by a von Mises plastic criterion and with both isotropic and kinematic hardening laws. The plastic criterion f at the mesoscopic scale is then defined in equation 3 by:

$$f = \frac{1}{2}\,(\underline{\underline{s}} - \underline{\underline{a}}) : (\underline{\underline{s}} - \underline{\underline{a}}) - (r + \sigma_y)^2 \qquad (3)$$

where $\underline{\underline{s}}$ is the deviatoric part of the stress tensor $\underline{\underline{\sigma}}$, $\underline{\underline{a}}$ is the stress tensor associated to the kinematic hardening, *r* is the isotropic hardening parameter and (σ_y is the initial yield limit.

$$\underline{\underline{a}} = c\,\underline{\underline{\varepsilon}}_p \qquad \dot{p} = g_p \tag{4}$$

with c, the hardening modulus, $p = \sqrt{\underline{\underline{\varepsilon}}_p : \underline{\underline{\varepsilon}}_p}$, the equivalent plastic strain rate. By taking into account the equation 2, this mesoscopic plastic criterion can be expressed in equation 5 by:

$$\begin{aligned} f &= \frac{1}{2}\left(\underline{\underline{S}} + \underline{\underline{p}} - \underline{\underline{a}}\right) : \left(\underline{\underline{S}} + \underline{\underline{p}} - \underline{\underline{a}}\right) - (r + \sigma_y)^2 \\ &= \frac{1}{2}\left(\underline{\underline{S}} - (2\,\mu + c)\,\underline{\underline{\varepsilon}}_p\right) : \left(\underline{\underline{S}} - (2\,\mu + c)\,\underline{\underline{\varepsilon}}_p\right) - (r + \sigma_y)^2 \end{aligned} \tag{5}$$

where $\underline{\underline{S}}$ is the deviatoric part of the macroscopic stress tensor $\underline{\underline{\Sigma}}$ (by definition, $\underline{\underline{\rho}}$ is already a deviatoric tensor).

Papadopoulos generalized the idea of fatigue limit, associating the fatigue damage and the elastic shakedown concept. As such: for any periodic loading,

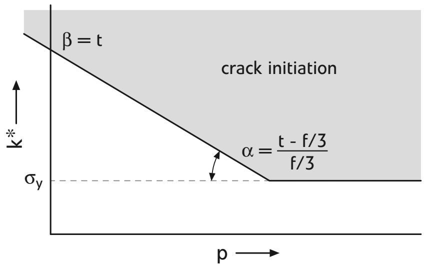

Figure 2. Graphical representation of the Papadopoulos HCF criterion. t and f are respectively the fatigue limit in torsion and in bending. σ_y is the initial yield limit in tension of the crystal

the limit of non initiation of cracks in a REV corresponds to the elastic shakedown limit, k_{lim}, at the mesoscopic scale of a misoriented grain. The grain is considered here as a crystal.

Let's suppose that $z = (2\,\mu + c)\,\underline{\underline{\varepsilon}}_p$. By considering the Melan's shakedown approach extended by Mandel [12,13], there is a shakedown state if

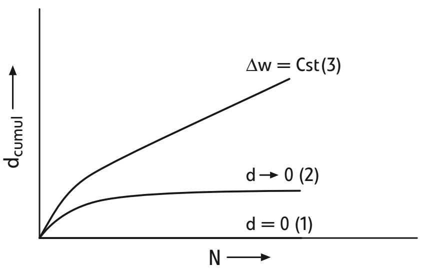

Figure 3. Theoretical evolution of the mesoscopic cumulated mechanical dissipation. The numbers in brackets indicate the three previous conclusions: elasticity, elastic shakedown and plastic shakedown

$$\forall t \geq t_0,\ \frac{1}{2}\left(\left(\underline{\underline{S}}(t) - z^*(t)\right) : \left(\underline{\underline{S}}(t) - z^*(t)\right)\right) < (r^* + \sigma_y)^2 \tag{6}$$

and in the case of r* and z* small. All the details concerning the determination of r* and z* are presented in Papadopoulos [2]. In the macroscopic deviatoric stress space, z* is the centre and $k^* = \sqrt{2}\,(r^* + \sigma_y)$ the radius of the smallest hypersphere circumscribed to the curve described by the tensor $\underline{\underline{S}}$ (t) which corresponds to the macroscopic loading path.

As a consequence of the preceding remarks the Papadopoulos fatigue criterion can be resumed as:

$$k^* + \alpha\, p_{max} \leq \beta \tag{7}$$

where p_{max} is the maximal mesoscopic hydrostatic pressure during the loading path. One can remark that p is equal to the macroscopic hydrostatic pressure P. The graphical representation of the criterion is presented on Figure 2.

Moreover, Papadopoulos [2] shows that:

$$k_{lim} = \frac{-\alpha\, p_m + \beta}{\alpha + \dfrac{k^*}{p_a}}\ \frac{k^*}{p_a} \tag{8}$$

where p_m and p_a are respectively the mean and the amplitude of the hydrostatic pressure and $k_{lim} = k_s$, the saturation stress of the grain as defined by Winter [11], now defined in a von Mises sense.

Dissipation and shakedown approach

Papadopoulos approach and mechanical dissipation. The mesoscopic rate of plastic work dp due to the plasticity in a misoriented grain can be defined in the previous framework as:

$$d_p = \underline{\underline{\sigma}} : \underline{\underline{\varepsilon}}_p \tag{9}$$

Considering the Papadopoulos approach, if $k^* \leq \sigma_y$, no plasticity occurs in such a grain. Therefore,

$$k^* \leq \sigma_y \Leftrightarrow d_p = 0 \tag{10}$$

If $k^* \leq k_{lim}$, the grain leads to an elastic shakedown state : there is no crack initiation. In this case, as recalled by Nguyen [14], the cumulated plastic work is bounded : d_p leads to zero with the number of loading cycles. Therefore,

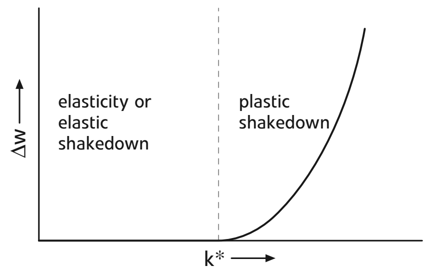

Figure 4. Theoretical evolution of the mesoscopic dissipated energy per cycle versus the macroscopic load (in the case of uniaxial loading)

$$k^* \leq k_{lim} \Leftrightarrow \lim_{N \to +\infty} d_p = 0 \text{ or } \int_0^{+\infty} d_p\, dt \leq +\infty \tag{11}$$

At last, if $k^* > k_{lim}$, the Papadopoulos criterion indicates that there is crack initiation and by considering the cumulated plastic strain as a damage indicator, Papadopoulos approach [2] allows to determine a number of cycles to crack initiation.

Halphen's shakedown theorem for elastoplastic structures with kinematic hardening [15] indicates that under a periodic loading, mechanical fields of such structures lead to a shakedown state. Therefore, if $k^* > k_{lim}$, isotropic hardening is saturated at k_s and, according to Halphen's theorem, after N_{ps} cycles, the misoriented grain leads to plastic shakedown where the cyclic plastic work Δw is constant. Then,

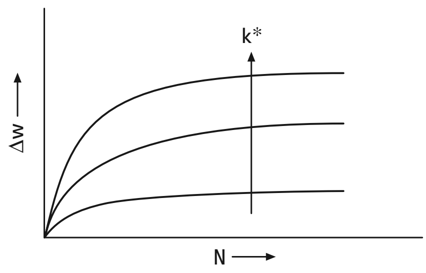

Figure 5. Theoretical evolution of the mesoscopic dissipated energy per cycle versus the number of test cycles (in the case of uniaxial loading)

$$\begin{aligned} &k^* > k_{lim} \text{ and } N > N_{ps} \Leftrightarrow \Delta w = \\ &= \int_{cycle} \underline{\underline{\sigma}} : \underline{\underline{\varepsilon}}_p\, dt = \int_{cycle} d_p\, dt = C^{st} \end{aligned} \tag{12}$$

The graphical expressions of the three conclusions (10), (11) and (12) are represented on Figures 3, 4 and 5. It now seems interesting to analyse the three conclusions in the framework of continuum thermodynamics.

Framework of continuum thermomechanics

In the Thermomechanics of the Irreversible Processes (TIP), the heat equation will couple locally the temperature field with the mechanical fields. By defining a free energy Ψ depending of the state variables α_j ($j = 1, 2, \ldots, n$) understood as mechanical fields, the heat equation can be written in the following form (see for example Besson at al. [16] or Lemaitre et Chaboche [17]):

$$\rho C \dot{T} = r + \mathrm{div}\,(\underline{\underline{K}}.\underline{\mathrm{grad}}\,(T)) + \\ + (\underline{\underline{\sigma}} : \underline{\underline{\dot{\varepsilon}}} + \rho T \frac{\partial^2 \Psi}{\partial T \partial \alpha_j} \dot{\alpha}_j - \rho \frac{\partial \Psi}{\partial \alpha_j} \dot{\alpha}_j) \quad (13)$$

where ρ denotes the density field, C the massic heat capacity, T the absolute temperature, r the distribution of heat sources, $\underline{\underline{K}}$ the thermal conductivity, $\underline{\underline{\varepsilon}}$ the mesoscopic strain tensor and $\underline{\underline{\sigma}}$ the mesoscopic stress tensor.

It is important to remark that this expression of the heat equation (13) is completely independent of the particular choice of a constitutive law. In the case of no coupling between the mechanical and thermal material behaviours, the previous expression is the classical heat equation:

$$\rho C \dot{T} = r + \mathrm{div}\,(\underline{\underline{K}}.\underline{\mathrm{grad}}\,(T)) + \underline{\underline{\sigma}} : \underline{\underline{\dot{\varepsilon}}} \quad (14)$$

In the case of the complete thermochanical coupling, the heat supply is decomposed in heat sources *r* and mechanical power:

$$\underline{\underline{\sigma}} : \underline{\underline{\dot{\varepsilon}}} - \rho \frac{\partial \Psi}{\partial \alpha_j} \dot{\alpha}_j + \rho T \frac{\partial^2 \Psi}{\partial T \partial \alpha_j} \dot{\alpha}_j \quad (15)$$

The mechanical power is naturally decomposed in a heat supply part, a stored part, and a thermomechanical coupling term: $\rho T \frac{\partial^2 \Psi}{\partial T \partial \alpha_j} \dot{\alpha}_j$

In the previous Papadopoulos mechanical background, the state variables are $\underline{\underline{\varepsilon}}_e$ and $\underline{\underline{\varepsilon}}_p$ and their associated thermodynamical forces are $\underline{\underline{\sigma}}$ the stress field and $\underline{\underline{a}}$ the stress tensor associated to the kinematic hardening. From a mathematical point of view, the thermodynamical forces are just the dual variables with respect to the part of mechanical work described.

By considering only a thermoelastic coupling (thermoplastic term is often negligible except in case of phase changes) and an additive decomposition of the strain tensor $\underline{\underline{\varepsilon}} = \underline{\underline{\varepsilon}}_e + \underline{\underline{\varepsilon}}_p$, the equation (13) becomes:

$$\rho C \dot{T} = r + \mathrm{div}\,(\underline{\underline{K}}.\underline{\mathrm{grad}}\,(T)) + \\ + \rho T \frac{\partial^2 \Psi}{\partial T \partial \underline{\underline{\varepsilon}}_e} \underline{\underline{\dot{\varepsilon}}}_e + \underline{\underline{\sigma}} : \underline{\underline{\dot{\varepsilon}}}_p - \underline{\underline{a}} : \underline{\underline{\dot{\varepsilon}}}_p \quad (16)$$

The first term on the right hand side in the last equation, r, is related to the existence of heat sources. The second term governs the transfer of heat by thermal conductivity. The third term corresponds to the thermoelastic effect which conducts to reversible conversion between mechanical and thermal energy.

One can finally recognize in the last terms on the right the rate of plastic work d_p and a term corresponding to a part of the dissipation stored in the hardening. Thus the intrinsic dissipation Φ is defined as:

$$\Phi = \underline{\underline{\sigma}} : \underline{\underline{\dot{\varepsilon}}}_p - \underline{\underline{a}} : \underline{\underline{\dot{\varepsilon}}}_p \quad (17)$$

Commonly, since the pioneering work of Taylor and Quinney [18], the ratio between the intrinsic dissipation Φ and the rate of plastic work $\underline{\underline{\sigma}} : \underline{\underline{\dot{\varepsilon}}}_p$ is often considered as a constant taking usually values between 0.8 and 0.9. That means that the stored part of the rate of plastic work is negligible.

Recent experiments performed at very high strain rates [19] show that this ratio is not constant, but depends on the loading.

To our knowledge such a comparison with experimental results is not available for fatigue loading. Therefore we will further consider, as a first approximation, that the stored part is still negligible compared to the rate of plastic work: $|\underline{\underline{\sigma}} : \underline{\underline{\dot{\varepsilon}}}_p| \gg |\underline{\underline{a}} : \underline{\underline{\dot{\varepsilon}}}_p|$. It results then that:

$$\rho C \dot{T} \approx r + \mathrm{div}\,(\underline{\underline{K}}.\underline{\mathrm{grad}}\,(T)) + \\ + \rho T \frac{\partial^2 \Psi}{\partial T \partial \underline{\underline{\varepsilon}}_e} \underline{\underline{\dot{\varepsilon}}}_e + \underline{\underline{\sigma}} : \underline{\underline{\dot{\varepsilon}}}_p \quad (18)$$

In the case of fatigue tests the external heat sources are absent and therefore $r = 0$.

To be able to solve without specific numerical recipes such a partial differential equation (PDE) (18), one must take assumptions on both last source terms. Therefore, we will, as a first approximation, neglect the thermoelastic effect and consider the plastic work constant and uniform on the specimen during a fatigue cycle (one can notice that the thermoelastic effect is reversible and the cumulative value of its generated heat on one fatigue cycle is equally nil,

$$\int_{cycle} \rho T \frac{\partial^2 \Psi}{\partial T \partial \underline{\underline{\varepsilon}}_e} \underline{\underline{\dot{\varepsilon}}}_e \, dt = 0.$$

Then, by denoting by *f* the test frequency, the equation (18) becomes

$$\rho C \dot{T} \approx \mathrm{div}\,(\underline{\underline{K}}.\underline{\mathrm{grad}}\,(T)) + \Delta w.f \quad (19)$$

where $\Delta w = \int_{cycle} \underline{\underline{\sigma}} : \underline{\underline{\dot{\varepsilon}}}_p \, dt$. By considering the one dimensional problem - i.e. the temperature field T depends only of x, the lengthwise coordinates and t, the time, - a 2L specimen's length, a uniform initial temperature $T(x,0) = T_0$ and boundary conditions such as $T(-L,t) = T(L,t) = T_0$, the previous PDE admits an asymptotic solution when equal to:

$$T(x = 0, t \to \infty) = T_0 + \frac{\Delta w.f.L^2}{2k} \quad (20)$$

In this case, the mesoscopic mechanical dissipation per cycle depends linearly on the temperature rise of the specimen, which can be indirectly measured by infrared thermography. Thus we have at the centre of such a specimen submitted to cyclic loading, in steady state:

$$\Delta w = \frac{2k.\Delta T}{fL^2} \quad (21)$$

Therefore, it is now possible to analyse the dissipated energy from experimental results.

Experimental results and discussion

Experimental results. Wang and Chen [20, 21] realized recently tensile low cycle fatigue tests on a Haynes HR120 superalloy with an analysis of the temperature variations observed by infrared thermography. Tests were performed at room temperature, with a 0.5 Hz loading frequency, and a strain ratio $R_\varepsilon = \frac{\varepsilon_{min}}{\varepsilon_{max}} = -1$. Figure 6 represents the temperature variations during tests realised at different strain range be-

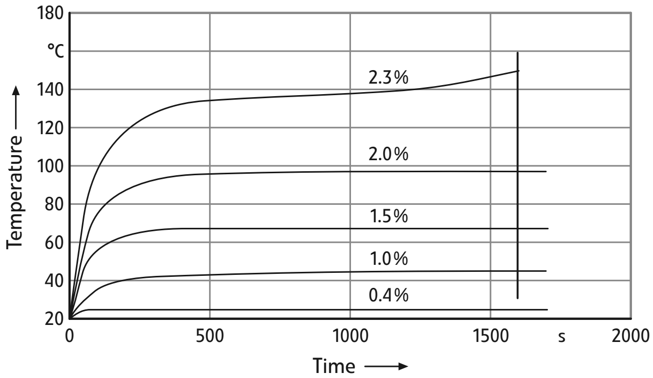

Figure 6. Experimental evolution of the temperature for tensile low cycle fatigue tests on HR120 superalloy after Wang's result [22]. One can remark the stabilized period after a fast increase of the temperature

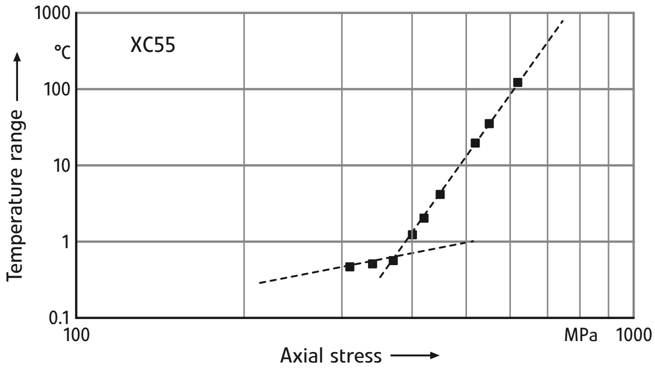

Figure 7. Evolution of the stabilized temperature versus the macroscopic load for a rotating bending test on an XC55 steel after Luong's results [27]

tween 0.4 % and 2.3 %. One can remark a stabilized level after a fast increase of the temperature. This stabilized temperature level depends naturally on the loading as previously shown by the equation (20). By taking into account the relation (21), one can calculate the corresponding dissipated energy per cycle Δw (k = 11.4 W °C^{-1} m^{-1}, f = 0.5 Hz and L_c = 17 10^{-3} m) for each loading condition. The length Lc corresponds to the length of the specimen including both gage and shoulder sections as discussed by Jiang et al. [22]. The results are presented on Table 1.

This stabilization of temperature is confirmed by results of La Rosa and Risitano [23] on a C70 steel tested under stress control at room temperature, with a 75 Hz loading frequency, and a stress ratio $R_\sigma = \sigma_{min}/\sigma_{max} = 0$. Some similar results are obtained on Ultimet superalloy by Jiang et al. [24, 24, 8] in high-cycle and low-cycle fatigue at room temperature. In this case, the HCF tests were realised at a frequency of 20 Hz and a ratio $R_\sigma = 0.05$. The LCF tests were realised at a frequency of 0.15 Hz and a ratio $R_\sigma = -1$.

Moreover, Luong [25] conducted rotating bending HCF tests at 100 Hz on a XC55 steel for different macroscopic loadings. Figure 7 presents the evolution of the stabilized temperature versus the different applied loadings. Recent results of Boulanger et al. obtained with tensile high-cycle fatigue tests on DP60 steel [26] confirm Luong's result.

It seems now interesting to compare computed dissipated energies per cycle coming from infrared measured temperatures by using the relation (21) with dissipated energies per cycle computed from hysteresis loops. In a previous work [27], some experimental LCF tests results coming from literature were synthesized in terms of dissipated energy per cycle. Those results correspond to different works: Chaboche et al. [7] on 316L steel with $R_\sigma = -1$, $R_\sigma = 0$ and $R_\sigma = -1$, Lieurade [28] on 35NCD16 steel with $R_\sigma = -1$ and Dias [9] on 80C4 steel with $R_\sigma = -1$. All those tests were realised at room temperature. Figure 8 presents the results by using the dissipated energy per cycle Δw as a damage indicator. For the 316L, the 35NCD16 and the 80C4 steels, Δw is directly obtained from the experimental hysteresis loop. For the Ultimet, the HR120 and the C70, Δw is obtained from the temperature measured by infrared thermography and thereafter from the relation (21).

Discussion

One can first compare Figure 6 corresponding to Wang's work [22] with the theoretical result presented on Figure 5: This stabilized temperature associated to the relation (21) show a plastic shakedown regime as defined in the theoretical part of this paper.

Moreover, the results obtained by Luong [27] are similar to the qualitative theoretical prediction displayed on Fig-

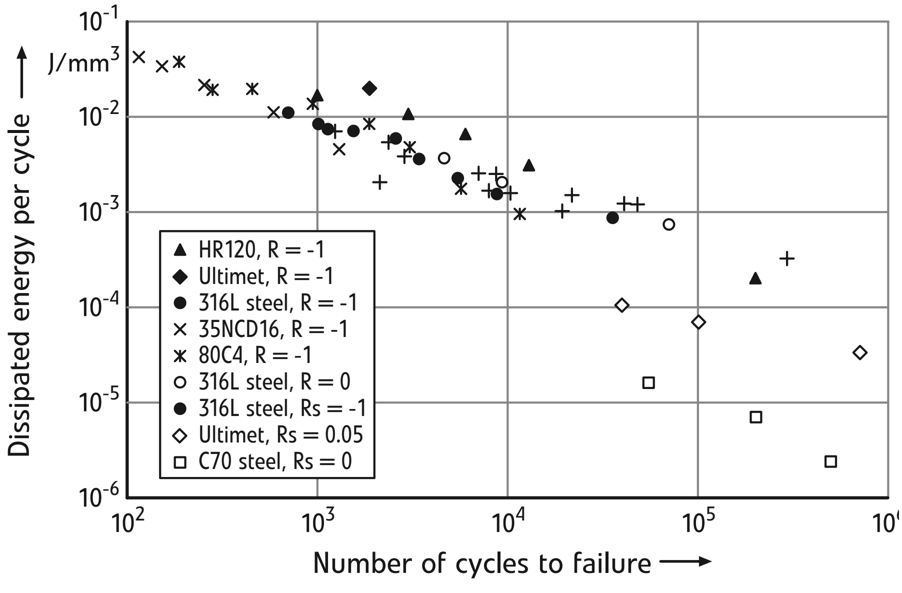

Figure 8. Comparison between dissipated energy obtained directly from the hysteresis loop (Chaboche et al. [7] for the 316L, Lieurade [30] for the 35NCD16, Dias [9] for the 80C4) or from infrared thermography measurements (Jiang et al. [24, 26, 8] for the Ultimet alloy, Wang, Chen et al. [22, 23] for the HR120 alloy and La Rosa and Risitano [25] for the C70). R corresponds to R_ε conditions and R_s to R_σ's one

ure 4. This result shows the link between the shakedown approach and the fatigue limit defined by the change in the dissipative rate.

Those both previous results allow to underline qualitatively the interest of such a dissipative approach coupled with shakedown concepts. In this way, the most important result is certainly the comparison presented on Figure 8. Indeed, all the energies obtained with the different materials and tests allow to obtain a quasi-unique curve which corresponds to a log-log relation between dissipated energy per cycle and the lifetime of specimens.

the principal way of progress. Indeed, one must remember that the area which is affected by cyclic plasticity does not always correspond to the whole specimen, in particular in the High Cycle Fatigue regime. Then, the hypothesis of a uniform constant heat source is certainly not the best assumption in the high-cycle regime. Thus, effects of the localisation have to be taken precisely into account and this justify the meso-macro approach in HCF as pointed out by Dang Van.

One must remember now that the stored part of the plastic work was neglected in this first approach. However,

ΔT (°C)	110	70	43	20	1.3
Δw (J.mm^{-3})	1.7 10^{-2}	1.1 10^{-2}	6.8 10^{-3}	3.2 10^{-3}	2.1 10^{-4}
N_f	1 000	3 000	6 000	13 000	200 000

Table 1. Temperature range, dissipated energy per cycle and lifetime obtained for the HR120 superalloy. The values of temperature and lifetime are taken from the graphics of Wang and Chen articles [22, 23]. The dissipated energy per cycle comes from the relation (21)

This relation was pointed out many years ago by Halford [29] and more recently by Charkaluk and Constantinescu in thermomechanical low-cycle fatigue [30]. Two experimental conditions do not verify this relation: the Ultimet superalloy at $R_\sigma = 0.05$ and the C70 steel at $R_\sigma = 0$. It is well known that a mean stress has an unfavourable influence on the lifetime in fatigue and a precise analysis of Jiang results [24] on Ultimet alloy shows moreover an initial ratchetting effect particularly damaging for the specimen in this conditions. This mean stress effect can explain the gap between those both experimental conditions with all the others even if it can be underlined that the slope remain approximately the same.

Moreover, this quantitative analysis show that values of dissipated energy per cycle obtained directly from the hysteresis loop or deduced from infrared thermography results are of the same order of magnitude even if the hypothesis used to solve the 1D heat coupled equation do not correspond exactly at the real situation.

An other aspect concerns the thermal hypothesis. Actually the exchange with the environment are totally neglected in this approach and have certainly to be taken into account in the analysis of the thermal measurements but this is not this part is certainly the most representative of the damage as plastic work is an upper-bound of this one. Then, one must take into account precisely the different thermomechanical terms in the equation (9). This justify an other approach in infrared thermography: the determination of heat sources from temperature measurements as realised by Louche [31] in plastic localisation problems. First attempts were recently made by Boulanger et al. [28] in fatigue context but the development of such an approach needs a lot of additional work.

Conclusion

In this paper, a dissipative framework based on shakedown concepts and also on the Dang Van's and Papadopoulos fatigue approaches was proposed. This framework allows to compare HCF and LCF results in terms of mechanical dissipation. The principal difficulty arises from the precise determination of this quantity from infrared thermographics measurements. More theoretical and experimental work is necessary to refine this approach. One of the great challenging goal is then the definition of a unified fatigue approach extensible to thermomechanical loadings [3].

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Summary

Dissipation und Ermüdungsschäden. Ermüdungsschäden werden generell in zwei Bereiche unterteilt: Ermüdung unter niedrigen Lastwechseln, Low Cycle Fatigue (LCF), und Ermüdung unter hohen Lastwechseln, High Cycle Fatigue (HCF). Der Übergangsbereich bezieht sich auf den Fall begrenzter Beständigkeit, während die unbegrenzte Beständigkeit generell mit HCF einhergeht. Ermüdungsschäden sind das Ergebnis mikroskopischer Wechselwirkungen, die unter zyklischer Beanspruchung auftreten. Der grundlegende Mechanismus, der in allen Lastwechselbereichen zur Rissinitiierung führt, liegt in der Ausweitung ineleastischer Dehnungen in den Körnern eines metallischen Werkstoffes infolge von Versetzungsbewegungen. Der Hauptunterschied zwischen HCF und LCF besteht darin, dass sich inelastische Dehnungen im mesoskopischen bzw. makroskopischen Bereich eines Werkstoffes ausbilden. Da die grundlegenden mechanischen Bedingungen dieselben sind, gibt es keinen Grund für unterschiedliche Kriterien für HCF und LCF. Der vorliegende Beitrag legt einige weiterführende Überlegungen, basierend auf der Energiedissipation in Richtung eines einheitlichen Verfahrens zur Behandlung von Ermüdungsschäden dar.