A unified approach for high and low cycle fatigue based on shakedown concepts

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ABSTRACT The purpose of this paper is to present a unified analysis to both high and low cycle fatigue based on shakedown theories and dissipated energy. The discussion starts with a presentation of the fatigue phenomena at different scales (microscopic, mesoscopic and macroscopic) and of the main shakedown theorems. A review of the Dang Van high cycle fatigue criterion shows that this criterion is essentially based on the hypothesis of elastic shakedown and can therefore be expressed as a bounded cumulated dissipated energy. In the low cycle fatigue regime, recent results by Skelton and Charkaluk *et al*. show that we can speak of a plastic shakedown at both mesoscopic and macroscopic scale and of a cumulated energy bounded by the failure energy. The ideas are also justified by infrared thermography tests permitting a direct determination of the fatigue limit.

Keywords dissipated energy; high cycle fatigue; low cycle fatigue; shakedown.

INTRODUCTION

Current industrial design is concerned with fatigue, as it is the main failure mode of mechanical structures under variable loadings. During a long period, starting with the pioneering work of Wöhler (1860) and ending in the late 1950s, high-cycle fatigue (HCF) was the most significant topic for engineers and researchers. They were mainly interested in establishing S–N curves (load versus number of cycles to failure) and in determining the fatigue limit for metallic materials. This approach is still used as a design tool in many cases to predict fatigue resistance of mechanical structures. In the 1960s, a special interest occurred in studying low-cycle fatigue (LCF). Instead of developing stress approaches, Coffin and Manson proposed fatigue models based on the strain amplitude or the plastic strain amplitude. The cited approaches showed their effectiveness for structures in the aeronautical and nuclear industry. However, success was obtained when the stress or the strain cycle was uniaxial and simple. This is not always the case, as stresses and strains are often multiaxial and present a complex path during the loading cycle. Application of classical fatigue models to mechanical structures is then difficult and the predicted results do not always match with test results.

Therefore, a sustained effort has been deployed for deriving reliable fatigue computational methods applicable to the different engineering configurations. The actual prediction techniques are generally based on multiaxial fatigue criteria using a stress approach in the HCF regime and on strain or inelastic strain approaches in the LCF domain. In spite of the fact that materials fail in both domains in similar manners, the two ways of modelling the fatigue problem seem so different that it is difficult to imagine any link between them.

The present paper discusses a series of criteria from an unified point of view in order to fill this theoretical gap. It is derived from Dang Van *et al*. HCF criteria[1,2] from some recent results in thermomechanical LCF[3,4] and shakedown theories.[5–8]

THE MULTISCALE APPROACH IN FATIGUE

In discussing fatigue phenomena we shall distinguish three scales:

- The *microscopic* scale of dislocations.
- The *mesoscopic* scale of grains.
- The *macroscopic* scale of engineering structures.

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In a simplified analysis we could say that the fatigue phenomena start generally with appearance of slip bands in grains, which broaden progressively during the cycles. The proportion of grains in which slip bands develop increases with the applied load.

The fatigue domains

Let us consider a structure submitted to a cyclic loading and let us represent the lifetime using a classical Wöhler diagram (Fig. 1). The lifetime should be understood as the initiation of the first visible crack at the macroscopic scale. For increasing loading levels, different fatigue phenomena will appear and contribute to this initiation; they can be classified as follows.

In the *high-cycle fatigue regime* (HCF), in general no irreversible deformation, i.e., plastic or viscous, is detected at the macroscopic level. The material behaviour seems to be purely elastic, meaning that stresses and strains are related through linear relations. However, at a *mesoscopic* level, irreversible strains occur in a certain number of grains and generate a *heterogenous* plastic strain field. Only misoriented crystals undergo plastic slip corresponding to a heterogeneous distribution of microcracks.

Within this region we can distinguish two domains. One at small loads where lifetime seems to be infinite denoted as *infinite endurance* (HCF-IE) and a second one, *finite endurance* (HCF-FE) at slightly higher load, where lifetime is large, but definitely finite.

The *low-cycle fatigue regime* (LCF) implies significant macroscopic deformations contributing to irreversible deformations. Now, stress and strain fields are generally related through highly nonlinear differential equations. At the mesoscopic level, the metal grains are subjected to plastic deformation in a more homogeneous manner than in the HCF regime. The first cracks at a mesoscopic level appear in the persistent slip bands quite early in the lifetime of the structure.

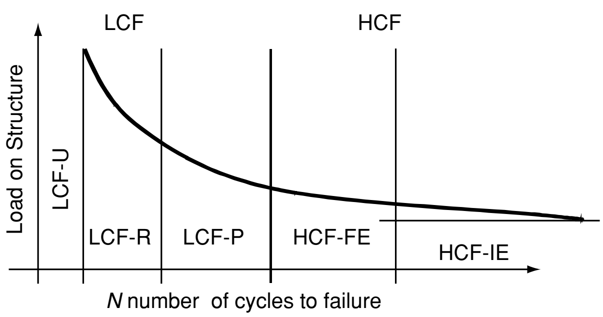

Fig. 1 A schematic Wöhler curve (load versus number of cycles to failure) and fatigue domains with increasing load.

Within this region, we generally distinguish three domains with increasing load, one where failure occurs due to alternating plasticity (LCF-P), a next one where failure occurs due to accumulation of plastic strain (ratchetting) (LCF-R) and finally a domain where structural collapse occurs directly from unrestricted plastic strain (LCF-U).

The meso-macro passage

In both LCF and HCF, damage phenomena occur in the grains, and therefore the use of mesoscopic fields seems to be relevant for studying these fatigue phenomena.

Let us recall, that in a certain sense the macroscopic fields (stress $\boldsymbol{\Sigma}$, strain $\boldsymbol{E}$, plastic strain $\boldsymbol{E}_p$,.) are related to the mean value of the mesoscopic fields (stress $\boldsymbol{\sigma}$, strain $\boldsymbol{\epsilon}$, plastic strain $\boldsymbol{\epsilon}_p$,...). In the theories of polycrystalline aggregates the macroscopic fields are therefore supposed constant in a small volume, surrounding the point under consideration, called 'representative volume element' or RVE. For instance the mesoscopic stress tensor $\boldsymbol{\sigma}$ and the macroscopic stress tensor $\boldsymbol{\Sigma}$ are related by the following formula:

$$\boldsymbol{\sigma} = \boldsymbol{A}\boldsymbol{\Sigma} + \boldsymbol{\rho} \quad (1)$$

where $\boldsymbol{\rho}$ is the mesoscopic residual stress and $\boldsymbol{A}$ is the fourth order tensor of elastic stress localization. In the case of equal mesoscopic and macroscopic elastic moduli, $\boldsymbol{A}$ is the identity tensor. The presence of the mesoscopic residual stresses (which depend closely on the characteristics of the loading path) shows that it is in general not correct to use the macroscopic stress $\boldsymbol{\Sigma}$ to characterize phenomena at the grain scale.

The evaluation of the local, mesoscopic fields from the macroscopic ones is in general a difficult task. *The material is locally heterogeneous and has to be considered as a complete structure when submitted to complex loading histories.*

Depending on the loading characteristics one can accept different simplifying assumptions, which will permit a solution to the problem. For load levels inducing an elastic macroscopic stress state, the relation (1) can be solved if $\boldsymbol{A}$ is known. For higher load levels, the macroscopic stress and strain state is elastoplastic and can be directly assimilated with the mesoscopic state, only if one assumes that all grains are equally deformed.

A simple direct measure of thermodynamically irreversible phenomena is the dissipated energy. Let us therefore, compare the dissipated energy at both mesoscopic and macroscopic scales. It is well known that the total macroscopic work rate is the mean value in the RVE of the local total work rate. However, the equality between the mesoscopic and macroscopic energy does not hold for plastic dissipation as proven by H.D. Bui and recalled in Ref. [1]. The difference between macroscopic plastic dissipation and mean value of mesoscopic plastic dissipation, decreases with increasing plastic strain, as the plastic heterogeneity from grain to grain decreases. This also justifies why macroscopic plastic deformation or plastic dissipation is a reasonable parameter in LCF.

CYCLIC BEHAVIOUR OF AN ELASTOPLASTIC STRUCTURE

In this section, we present a series of mathematical results regarding the elastoplastic behaviour of a structure submitted to cyclic loadings. As such, the following results are completely independent of the microstructure.

The main question concerns the existence of an asymptotic response of the internal variables ($\boldsymbol{\Sigma}$, $\boldsymbol{E}$, $\boldsymbol{E}_{\mathrm{p}}$,...).

We can classify these responses as follows and relate them directly with the mechanical behaviour observed at a macroscopic scale described in The meso-macro passage section.

1 *Existence of a limit cycle*

- Alternating elasticity (elastic shakedown) (in HCF-IE and HCF-FE).
- Alternating plastification (plastic shakedown) (in HCF-FE and LCF-P).

2 *No limit cycle*

- Ratchetting, i.e., accumulation of plastic strain over all cycles (in LCF-R).
- Unrestricted plastic yield (in LCF-U).

The asymptotic response of a structure under a cyclic loading can be determined after a complete computation of a large number of cycles or can be directly estimated through application of the theoretical results recalled in the next section.

Direct analysis of structures

The theories and methods for the direct analysis of structures, i.e., estimating directly the limit cycle, provide the 'live load' multiplier also denoted as a 'safety factor' characterizing the limit between the elastic and the plastic shakedown domain of the structure. The techniques used for this purpose are based on a kinematic or a static approach and we shall only present the main results in order to highlight their relation to the underlying fatigue problems.

The limit between the plastic shakedown and the ratchetting regions has not been extensively studied. In spite of recent successes in computing directly the limit elastoplastic cycle[9, 10] we still do not have complete theoretical results characterizing this limit.

The original static theorem of Melan gives a sufficient condition for elastic shakedown for a structure made of elastic perfectly plastic material submitted to cyclic loading.

Melan's Theorem: *If there exist a time instant T and a time independent self-equilibrated residual stress field $\boldsymbol{R}(x)$ and a safety factor $m > 1$ such that for all points x of the structure and for $t > T$ the following inequality holds*:

$$g\,(m\boldsymbol{\Sigma}_{\mathrm{el}}(x,t) + \boldsymbol{R}(x)) - k^2 \leq 0$$

then the structure will shakedown elastically.

Here g and k denote, respectively, the yield criterion and the yield limit. $\boldsymbol{\Sigma}_{\mathrm{el}}$ is the purely elastic stress response of the structure under the same external loading.

Without getting in the details of the precise mathematical proof let us recall that two main steps are needed. In the first one, it is proven that under the hypothesis of the theorem, the dissipated energy is bounded. In the second step, it is shown that the distance between the real stress $\boldsymbol{\Sigma}(x,t)$ and the purely elastic estimation $\boldsymbol{\Sigma}_{\mathrm{el}}(x,t)$ tends toward a constant value. The key point of the proof is the *associative* property of the plastic model, i.e., the fact that *the plastic flow is normal to the convex plasticity domain*.

The results do not state anything about the magnitude of the plastic deformation before shakedown, but one can extract the idea of a bounded plastic work in order to ensure a bounded plastic strain, as explained by Koiter:[5] "*if the total amount of plastic work performed in the loading process is accepted as suitable criterion for assessing the overall deformation, boundedness of the overall deformation may be proven if the structure has a safety factor $m > 1$ with respect to shakedown*."

Melan's shakedown theorem has been extended by different authors to account for more realistic material behaviour. For isotropic and kinematic hardening material, Mandel *et al*.[7] propose another formulation which is a *necessary* condition of elastic shakedown:

Mandel's Theorem: *If there exist a time T and a time independent (fixed) stress field $\boldsymbol{\Sigma}^*$ such as for all x and $t > T$*:

$$g\,(\boldsymbol{\Sigma}_{\mathrm{el}}(x,t) - \boldsymbol{\Sigma}^*(x)) - \boldsymbol{K}^{*2}(\boldsymbol{E}^{\mathrm{p}}_{\mathrm{eq}}(x)) \leq 0$$

the structure will shakedown elastically.

The isotropic hardening parameter K, also interpreted as a yield radius, is an increasing function of cumulated equivalent plastic strain $\boldsymbol{E}^{\mathrm{p}}_{\mathrm{eq}}$ and $\boldsymbol{K}^*(\boldsymbol{E}^{\mathrm{p}}_{\mathrm{eq}})$ is the maximum acceptable value of the yield radius before failure.

In fact, as the yield criterion depends only on the stress deviator, only the deviatoric part intervenes in the previous inequality. At shakedown, the deviators of $\boldsymbol{\Sigma}^*$ and $\boldsymbol{K}^*(\boldsymbol{E}^{\mathrm{p}}_{\mathrm{eq}})$ are, respectively, the centre and the radius of the smallest hypersphere surrounding the loading path defined by the deviator of $\boldsymbol{\Sigma}_{\mathrm{el}}(x,t)$.

Koiter's reasoning can be also extended to strain hardening materials using the framework of generalized standard materials as introduced by Halphen and Nguyen (see for instance Ref. [8]). This concept recovers a wide class of elastoplastic or elastoviscoplastic materials including classical isotropic and kinematic hardening materials. Beside the usual strain parameters $\boldsymbol{E}$, $\boldsymbol{E}_{\mathrm{p}}$ Nguyen *et al*. introduced internal parameters including plastic deformation and a set of strain hardening variables denoted symbolically by $\boldsymbol{\beta}$. It is then possible to define a potential energy $\boldsymbol{W}(\boldsymbol{E}, \boldsymbol{E}_{\mathrm{p}}, \boldsymbol{\beta})$ such as:

$$\boldsymbol{\Sigma} = \frac{\partial \boldsymbol{W}}{\partial \boldsymbol{E}}, \quad A_{\mathrm{p}} = -\frac{\partial \boldsymbol{W}}{\partial \boldsymbol{E}_{\mathrm{p}}}, \quad A_{\beta} = -\frac{\partial \boldsymbol{W}}{\partial \boldsymbol{\beta}}$$

The elastic domain is a convex C in the space of generalized forces, defined by:

$$g(\boldsymbol{\Sigma}, \boldsymbol{A}_{\mathrm{p}}, \boldsymbol{A}_{\beta}) \leq 0$$

The generalized stresses $\boldsymbol{A} = (\boldsymbol{\Sigma}, \boldsymbol{A}_{\mathrm{p}}, \boldsymbol{A}_{\beta})$ are said to be plastically admissible if they satisfy the last inequality. As in perfect plasticity, the normality flow rule is verified by the generalized internal variables. The static shakedown theorem for standard generalized materials is formulated as follows:[8]

Q.S. Nguyen's Theorem: *Elastic shakedown occurs, for all initial states of the structure, if there exist a field of internal parameters* ($\boldsymbol{E}_{\mathrm{p}}^{*}$, $\boldsymbol{\beta}^{*}$) *and a security coefficient* $m > 1$ *such that the associated field of generalized forces* $m\boldsymbol{A}^{*}(t)$, *obtained with fixed internal parameters* ($\boldsymbol{E}_{\mathrm{p}}^{*}$, $\boldsymbol{\beta}^{*}$), *is plastically admissible whenever* $t > T$.

In this case the boundedness of the dissipation, ensures that the cumulated plastic deformation and strain hardening parameters are bounded.

In the previous discussion we focused only on results based on a static approach. Let us mention that the dissipated energy is equally a key point of the dual results based on a kinematic approach.[5] More precisely, in a general framework one can admit that the structure will shakedown if the internal plastic work computed for all admissible plastic strain rate cycles is always greater than the upper bound of the work computed from all external loads.

SHAKEDOWN ANALYSIS OF FATIGUE

We propose to use the shakedown theory for the behaviour of the microstructure in order to derive a unified model valid for structural applications from high to low-cycle fatigue.

High cycle fatigue – application to fatigue limit

In the high-cycle regime, only few grains, badly orientated with respect to the loading, undergo plastic deformation in localized slip bands. Under the fatigue limit, i.e., the threshold between the infinite and finite lifetime, the dissipation at the mesoscopic scale is bounded and therefore the dissipated energy per cycle decreases to become after a while negligible (Fig. 2). On the same figure, point A represents the elastic shakedown limit. Beyond the point A, plastic shakedown occurs which will lead to fatigue failure characterized by the decrease of the stress. The corresponding dissipated energy is bounded. We already pointed out the difficulties to relate the energy dissipation at mesoscopic and macroscopic scale.

In engineering applications, it is therefore not easy to characterize this energy by evaluating directly the accumulated plastic strain, and consequently the dissipated energy cycle by cycle as was done by Papadopoulos.[2,11] This calculation is rather a theoretical model of fatigue than a practical proposal for industrial applications. It is the reason why two different ways were explored to derive high cycle fatigue criterion, based on the use of stabilized stress parameters instead of dissipated energy.

When the loading characteristics correspond to the fatigue limit, the asymptotic stabilized stress state is contained in a yield limit surface defined by the limit radius K^{*}. This stabilized state corresponds to point A of Fig. 2.

Dang Van[1] generalizing an idea of Orowan, proposed to consider the mesoscopic current stress state at the stabilized (shakedown) state as the relevant parameter in order to formulate a high-cycle multiaxial fatigue resistance criterion. More precisely, the proposed criterion is a combination of the mesoscopic shear $\tau(t)$ and the simultaneous hydrostatic pressure $p_{\mathrm{H}}(t)$. It can be stated as follows:

For all instant t beyond T, one has $\tau(t) + ap_{\mathrm{H}}(t) - b < 0$, fatigue will not occur, where a and b denote material constants which can be determined by two simple fatigue

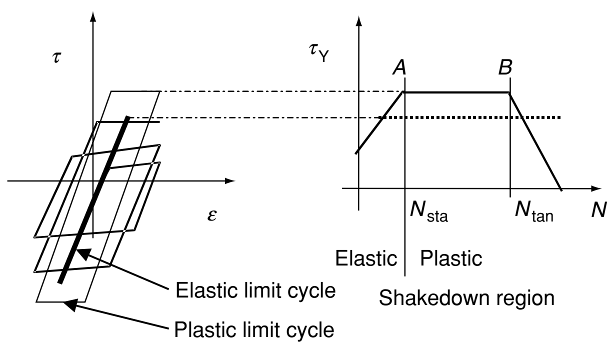

Fig. 2 Elastic and plastic shakedown at the mesoscopic scale in HCF: the mesoscopic behaviour of the RVE and the limit cycles (left) and their projection on the cyclic behaviour of the yield limit of a slip plane (right).

experiments on classical test specimens. For instance *b*, corresponds to the fatigue limit in simple shear.

Papadopoulos proposed a theory in which K^* depends (linearly) on the maximum hydrostatic tension induced by the loading cycle.[14]

The general application of this criterion requires (see for details Ref. [1]):

- First to evaluate the mesoscopic stress tensor knowing the macroscopic stress cycle; this can be achieved by application of the previously recalled shakedown theorems in particular Mandel and Nguyen's theorem. For this purpose, one constructs the smallest hypersphere surrounding the macroscopic deviatoric stress-loading path, the centre of which defines (approximately) the local deviatoric residual stress $\boldsymbol{\rho}$. Then the mesoscopic stress is known at any time of the loading cycle.
- Secondly, one must consider the plane and the instant for which the set ($\tau(t)$, $p_H(t)$) is a 'maximum' relative to the criterion.

Details of the resulting computational method and some examples of applications can be found in Ref. [1].

This general framework is also used to derive a high-cycle anisothermal fatigue criterion with temperature-dependent fatigue limits[12] and to predict limited lifetime in high-cycle fatigue of welded structures.[13]

Low cycle fatigue – cyclic material behaviour

The low cycle fatigue regime is characterized by extended irreversible plastic or viscous deformations. A direct computation of the mesoscopic strain from the macroscopic strain is not possible due to the material non-linearity.

In this case let us analyse the typical cyclic material behaviour in a strain-controlled experiment, expressed through the macroscopic stress evolution as a function of the number of cycles (Fig. 3).

The apparent load evolves through 3 stages defined by the *stabilization*, the *tangent*, and the *final* points. The first phase corresponds to a cyclic hardening or softening. During this period the microstructure is subjected to plastic loading and converges to a plastic shakedown limit cycle generally attained at N_{sta} cycles. At this point, it is generally accepted that the first microcracks with a length scale of several μm appear.[14]

After the initial *stabilization* phase, the material behaviour is stable and evolves. The microscopic crack grows and attains a macroscopic length after N_{tan} cycles. After this cycle, the crack influences the macroscopic fields of the structure and the applied load is decreasing with growing crack length. Failure is now to be expected after a small number of cycles at N_{fin}.

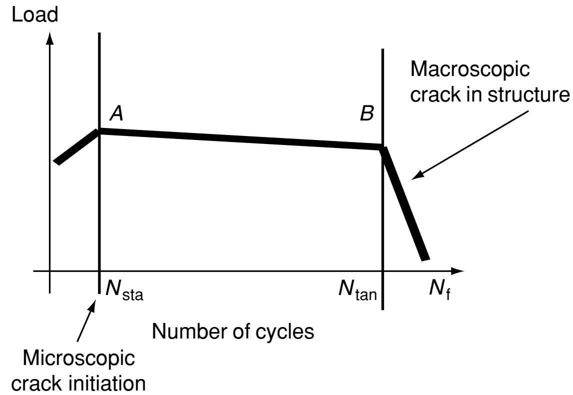

Fig. 3 The cyclic material behaviour at a macroscopic level.

One can remark that the macroscopic cyclic behaviour (Fig. 3) is very similar to the mesoscopic cyclic behaviour of the grain in HCF regime (Fig. 2) discussed previously. This can be explained by that fact that with increasing irreversible strain, the mesoscopic strain becomes more homogeneous and thus closer to the macroscopic state.

The endurance criterion in the HCF regime, assumes that the cyclic evolution of the internal state at a mesoscopic level is stopped since an elastic shakedown state is reached before the point A (see Fig. 2).

In the LCF regime, the corresponding point A at N_{sta} cycle in the cyclic material behaviour (see Fig. 3), describes the point where the macroscopic state converges towards a plastic shakedown regime. At the mesoscopic level, the material evolves also in a plastic shakedown cycle (Fig. 2).

A short analysis of the classical LCF criteria shows that these are based on the stabilized material behaviour in a plastic shakedown regime, which characterizes the major part of the lifetime during the evolution from the N_{sta} to the N_{tan} cycle. Therefore, these criteria are focused on different parameters characterizing the dimensions of the cycle in order to define the evolution of damage.

In order to distinguish the cases where the strain rate plays an important role, a major trend was to differentiate plastic and creep fatigue. However, in both cases fatigue phenomena are associated with irreversible strain fields and therefore directly related to the energy dissipation.

An inspection of the initial work of Manson and Coffin shows that they considered the amplitude of plastic strain as the relevant parameter for fatigue. Their tests were uniaxial and strain controlled. In this case the stress is related to the plastic strain amplitude for the stabilized cycle and therefore the plastic dissipation can be considered directly as function of strain amplitude. However, in case of more complex loading cycles conducting to multiaxial stress/strain paths there is no simple relation between plastic strain and dissipated energy.

The idea of dissipated energy per cycle, obtained from following integral:

$$W = \int_{\text{cycle}} \sigma \, d\boldsymbol{\epsilon}$$

has been appealing and has been involved in a series of classical criteria like Morrow, Garud, Ostergren, Ellyin *et al.*, Bui Quoc *et al.*, ... (see Socie and Marquis[15] and Suresh[16] for a review). In these criteria one can find a direct reference to energy dissipation through the interpretation of the fatigue factor involving products of strain and stress. Unfortunately, most of these criteria are not intrinsic and cannot therefore be generalized to multiaxial situations.

The more or less *tacit* assumption of the existence of a stabilized macroscopic cycle has been skipped in the last decades. For applications related with nuclear or aeronautical structures, a series of precise elastoplastic or elastoviscoplastic constitutive equations coupled with damage evolution were proposed by Lemaitre and Chaboche.[17] Without discussing in detail these proposals let us just mention that the computation of all cycles during the lifetime of an engineering structure can easily become cumbersome and contribute to extremely high computational times.

Next we shall reintroduce the assumption of a stabilized cycle. The reasoning will afterwards be based on an interesting interpretation of the fatigue life using the cyclic material behaviour recently proposed by Skelton.[14] The experimental evidence collected from a series of isothermal and anisothermal LCF tests on steels show that the cumulated dissipated energy to the saturation point, i.e., during the first N_{sta} cycles, is a constant $W_{sta} = 1\text{–}10\,\text{J mm}^{-3}$. One can interpret these results as a mesoscopic crack initiation criteria or equivalently as the failure of the mesoscopic elementary volume. It is the failure of the same RVE discussed in the Dang Van HCF criterion. However, the endurance limit (HCF-IE) is defined as the maximal load that avoids the failure of the RVE.

The fact that the cumulated dissipated energy W_{sta} is constant, has also been justified by complementary work of Skelton estimating the dissipated energy for the failure of an elementary volume in the crack propagation process.

In conclusion one can interpret W_{sta} as an upper bound for the dissipated energy to *elastic shakedown* in HCF and a lower bound for the *plastic shakedown* in LCF.

Starting from this remark and from the fact that the dissipated energy per cycle is an intrinsic quantity which can be summed up under complex anisothermal loading conditions and computed from multiaxial strain and stress fields, Charkaluk and Constantinescu[3] proposed an interpretation of the fatigue life of complex structures. Some applications of this criterion, i.e., lifetime predictions of exhaust manifolds and cylinder heads, are presented in Ref. [4]. Let us just summarize here that the discussed problems involve anisothermal loading creating a multiaxial strain–stress path. The subsequent fatigue problems include creep and plasticity. They have considered that the material behaviour is stabilized after a small number of cycles and the global behaviour of the structure is computed directly in this configuration. The dissipated energy per cycle, characterizes the fatigue evolution during the lifetime. It has been shown that the dissipated energy per cycle can be used to predict the failure of the macroscopic representative volume element of specimens and structures. The prediction can be expressed as an upper bound on the maximum admissible energy dissipation during the lifetime of the structure. The importance of the dissipated energy per cycle stems from the possibility to add up irreversible phenomena at low temperature (where material behaviour and damage are characterized by macroscopic plasticity) with phenomena at high temperature (where damage phenomena and material behaviour are characterized by creep, i.e. viscosity). In these cases, classical approaches are not effective since stresses vary with temperature and plastic strains.

During the previous analysis, we have combined two main ideas: the existence of a shakedown state and the boundedness of the dissipated energy to failure.

Experimental evidence by infrared thermography

When submitted to HCF, components are elastic at a macroscopic level and dissipation can occur only in some isolated grains at the mesocopic level as already discussed before. Some researchers have shown, that the underlying dissipation phenomena can be captured using infrared thermography. A direct consequence of these results is a rapid evaluation of the HCF endurance limit, instead of using the conventional stair case method.

Infrared thermography is a convenient technique for producing temperature images obtained from the invisible radiant energy emitted by the test specimen submitted to cyclic loading at an adequate frequency. The temperature rise can be thus captured by a thermographic camera. When the load is increased, one can observe simultaneously a temperature increase. As it was shown by Luong *et al.*[18–20] (Fig. 4), the fatigue damage is revealed by a sudden change of the rate of the dissipated heat which is proportional to the rate of mechanical dissipation at a mesoscopic level. The fatigue limit (HCF endurance limit) is represented by the point of rate change. It is therefore obtained instantaneously by this

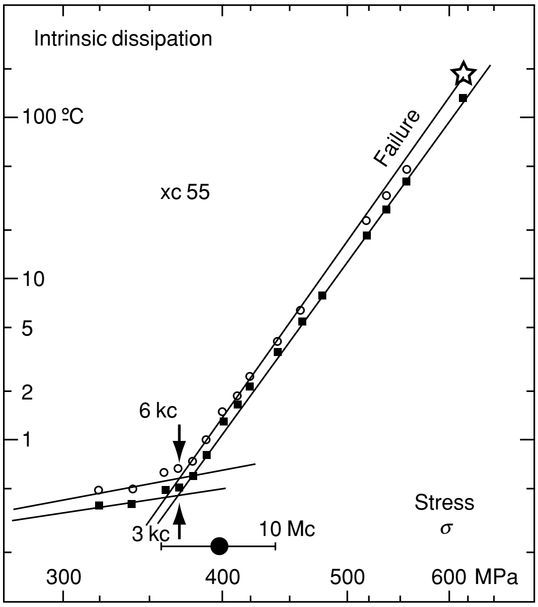

Fig. 4 Variation of the rate change in dissipated energy with the increase of the stress in rotating bending for XC55 steel. The HCF endurance limit coincides with the sudden increase of the rate of dissipation. The measurements have been done after 3000 and 6000 cycles at 50 Hz which assures that a shakedown state has been attained (after Luong).[20]

method using infrared thermography and the results match those obtained in conventional fatigue tests.

Similar sudden changes in the dissipation regime were previously reported by researchers in different countries from observations using other techniques.[21,22] Instead of measuring directly the temperature increase in the stabilized state, these authors introduced the concept of internal friction or specific damping parameter which are not so well defined. Regarding the point of sudden change of the rate, Lazan[21] called it referring to damping increase: 'the cyclic stress sensitivity limit'. When comparing these points with the fatigue limit, we remark that they lie lower than the fatigue limit. This can be explained by the fact that these points stem from global mechanical measurements over the specimen and refer therefore to a mean of the thermomechanical fields, whereas the infrared thermography provides a local measurement conducting to an accurate rate evaluation related to the critical regions.

It is important to remark, that in spite of the industrial success in applications we still do dispose of as convincing theoretical explanation of the coincidence between the fatigue limit and the point of sudden rate change in thermal dissipation. Intuitively using the present approach for interpreting fatigue phenomena the reason of this coincidence is straightforward. The fatigue limit corresponds to a threshold on dissipative energy between elastic and plastic shakedown at a mesoscopic level. Below this threshold, dissipation (which increases with increasing stress level) is bounded, and as a consequence plastic strain and internal strain hardening parameters are finite. Above the threshold energy is dissipated at each cycle and the upper limit of the cumulative dissipated energy is attained at failure after a finite number of cycles. This has also been proven by the previous shakedown results of Koiter[5] for perfectly plastic materials and extended by Nguyen[8] for generalized standard material.

CONCLUSION

Metal fatigue has been during the last one and a half century the subject of numerous research studies conducted by scientists of different fields: mechanical engineers, material scientists, physicists, chemists. Although progress can be reported in the understanding of the underlying physical phenomena, a series of difficulties still exist to achieve an interdisciplinary consensus of the way to model fatigue crack initiation. This paper presented an overview of the field showing that the main assumptions can be reduced to *shakedown theories* and to the *bounding of the dissipated energy* to failure. The discussions are based on examples of industrial structures submitted to complex multiaxial loading which have successfully been solved in the domain of high-cycle fatigue regime as well as low-cycle in plastic or visco-plastic creep fatigue regime.[1–4, 11,13]

As a final conclusion, let us recall this sentence written in 1963 by Drucker which summarizes our proposal:[23] 'when applied to the microstructure there is a hope that the concepts of endurance limit and shakedown are related, and that fatigue failure can be related to energy dissipated in idealized material when shakedown does not occur'.

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